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PHYSICAL MEASUREMENTS.

A

COURSE OF EXPERIMENTS

IN

PHYSICAL MEASUREMENT.

In Four Parts.

STATE NORMAL SCHOOL
LOS ANGELES, CAL.

PART II.

SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

3462

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PHYSICAL MEASUREMENT.

Part Second.

MEASUREMENTS IN SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

SOUND — Continued.

EXPERIMENT LI.

VELOCITY OF SOUND.

¶ 135. **Determination of the Velocity of Sound.** — (1) Two data are required for the determination of the velocity with which sound passes from one point to another: 1st, the distance between two stations (see ¶ 136); and 2d, the time occupied in traversing this distance (see ¶ 137). To make use of the results, the temperature of the air must be found at various points between the two stations (see Part I. ¶ 15); and if precision is required, the humidity of the air should also be determined.[1] The velocity of sound is not affected by barometric pressure.

---

[1] At ordinary summer temperatures (20° to 30°) the effect of humidity upon the velocity of sound may amount to one half of 1 %. See Table 15, B.

(2) If the path traversed by the sound is at right-angles with the direction of the wind, the velocity of sound will not be perceptibly affected by any ordinary atmospheric disturbance. It is, however, increased by the velocity of the wind when the two move in the same direction, or diminished by the same amount when they move in opposite directions.[1] When the directions are oblique, the velocity of sound is always more or less affected. It is therefore best to arrange an experiment so as to find the time occupied by sound in traversing a given distance first in one, then in the opposite direction. In this case, if the velocity of the wind is small and tolerably constant, the *average* result will not be perceptibly affected by it.

(3) Two or more determinations of the velocity of sound should be made between stations at different distances. Any constant error in the estimation either of distance or of time will be shown by a disagreement of the several results. The true velocity of sound is to be calculated in such a case from the *difference in time* required to traverse two given distances (see formula II. below).

(4) Let d be the distance traversed by sound in the time t; then the velocity of sound, v, is to be calculated by the equation

$$v = \frac{d}{t}. \qquad \text{I.}$$

---

[1] A velocity of the wind amounting to 10 metres per second, or about 22 miles per hour, would affect the velocity of sound by about 3 %.

Distinguishing by subscript numerals 1 and 2 the results in the two cases, we should have

$$v = \frac{d_1}{t_1} = \frac{d_2}{t_2};$$

hence,

$$\frac{d_1}{d_2} = \frac{t_1}{t_2}.$$

Subtracting 1 from both sides of the equation we have

$$\frac{d_1}{d_2} - 1 = \frac{t_1}{t_2} - 1;$$

or, reducing to a common denominator,

$$\frac{d_1 - d_2}{d_2} = \frac{t_1 - t_2}{t_2};$$

whence

$$\frac{d_1 - d_2}{t_1 - t_2} = \frac{d_2}{t_2}.$$

Finally, substituting equals for equals, we find

$$v = \frac{d_1 - d_2}{t_1 - t_2}. \qquad \text{II.}$$

By the use of this formula, constant errors (§ 24) are eliminated.

¶ 136. **Measurement of Long Distances.** — The measurement of long terrestrial distances is in general a problem for which the student must be referred to works on surveying. No particular difficulty will, however, be found in measuring approximately a distance along a moderately straight path; for even variations as great as 8° (nearly 1 foot in 7), either in the direction or in the slope of the path, will introduce an error of less than one per cent in the result.

Distances may also be determined indirectly by means of a sextant. To measure a distance, for example, across a valley, from an observing station, *A*, (Fig. 123) to an object *B*, we place (or select) an object *C*, so that the lines joining *B* with *A* and with *C* may be approximately at right-angles. The distance *BC* is then measured directly, and the angle *CAB* is determined from the observing station. Since (by definition)

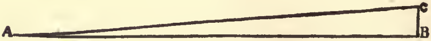

Fig. 123.

$$BC \div AB = tangent\ CAB,$$

we have

$$AB = \frac{BC}{tan\ CAB}.$$

To obtain with an ordinary sextant (see ¶ 124) results accurate within 1 per cent, the distance *BC* actually measured should be at least a hundredth part as great as the distance *AB* to be determined. In regard to the direction of *C* from *B*, great accuracy is not required. If the corner of a square be placed at *B* (Fig. 124) with one side directed towards *A*, any object, *C*, nearly in range with the other side of the square, will answer for our purpose. An error of 8° in the angle *ABC* will introduce an error of only 1 % in the result. The object *C* may

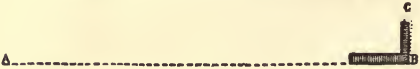

Fig. 124.

be on a level with *B* or above it, as may be more convenient. The distance *BC* and the angle *CAB* must be accurately measured.

In one part of the experiment the distance *AB* should be as great as possible considering the space at the disposition of the observer, and the distance through which the signals at his command can be seen or heard. If the method of difference is to be employed (¶ 135, 3), it is necessary, in a second part of the experiment, to make use of a much shorter distance. The second distance should be in no case greater than half of the first, and always as small as is consistent with the accurate determination of the time occupied by sound in traversing it. When the time is to be found by an ordinary watch (¶ 137, I.), the smaller distance should be several hundred, the greater several thousand metres. In the pendulum method (¶ 137, IV.), distances of 300, 600, and 900 metres may conveniently be employed. When sound signals are to be sent back and forth between two stations (¶ 137, III.), the minimum distance may be reduced to about 150 metres. The velocity of sound has been determined by the use of echoes (¶ 137, II.) between the Jefferson Physical Laboratory and the Lawrence Scientific School, the walls of which are about 80 metres apart. Long corridors, tunnels, and conduits of various sorts frequently give rise to echoes suitable for the determination of the velocity of sound.

It must be remembered that in the time required for a signal to go from one station to another, then back to the first, the distance traversed is twice that between the stations. When the sound is reflected back to the observer the distance traversed is twice that of the observer from the object causing the reflection. Care must be taken to identify the object in question. In the interval between two successive echoes, sound must obviously traverse twice the distance between two objects which reflect it, as for instance two parallel walls or the two ends of a conduit.

¶ 137. **Measurement of Short Intervals of Time.** — I. One of the oldest methods of estimating the time required for sound to traverse a given distance is to count the ticks of a watch which occur between the flash and the report of a cannon discharged at that distance from the observer (see ¶ 138). When, owing to obstructions in the field of view, it is impossible to see the flash, an electric telegraph may serve in the place of light to inform the observer of the exact moment of the discharge.[1] Instead of counting ticks, a "stop-watch" may be used, or a chronograph may be employed (¶ 266). Amongst various ingenious devices for the measurement of small intervals of time may be mentioned the use of a stream of mercury from a Mariotte's bottle (see Fig. 275, ¶ 250), which may be directed into a receptacle at the beginning of the interval, and diverted at the end of the interval. The quantity of mercury collected serves to estimate very precisely the interval of time in question.

---

[1] The velocity of light is about 30,000,000,000 *cm. per sec.*; hence the time lost in traversing terrestrial distances may generally be disregarded. An electric current is practically instantaneous in its action; but an allowance must be made for the slowness of telegraphic instruments to respond to the current, unless a method of difference be employed. See ¶ 135, 3.

II. In certain localities the velocity of sound may be similarly determined by timing the interval between a sound and its echo. When a series of echoes may be heard, the interval between them may be determined by adjusting a pendulum or a metronome so as to keep time with the echoes while they last, then afterward finding the rate of the pendulum or metronome, by timing 100 or more oscillations. Again, a method of multiplication may be used (§ 39). When the last audible echo reaches the observer, a new sound may be made; so that the interval of time to be measured may be indefinitely increased. One of the earliest determinations of the velocity of sound is said to have been made by a monk, who made use of the echo in a cloister caused by clapping his hands. The sounds thus produced were, it is said, so timed as to alternate regularly with the echoes.

III. The effects of an echo may be imitated by a series of sound signals interchanged between two stations. Let us suppose that two observers, each provided with a hammer and a plank, place themselves at suitable distances (see ¶ 136). The first gives a blow with his hammer, then the second returns the signal as soon as the sound reaches him. When the first hears the response, he gives another blow, etc. As in the last method (II.), the interval of time to be measured may be indefinitely multiplied.

With practice, each observer will learn to anticipate the return signal, so that very little time will be lost in the act of repetition. The time thus lost is to be eliminated by making two experiments, as has been suggested above (¶ 135, 3).

IV. Another method[1] is to station two observers let us say 300 or 350 metres apart, and to provide each with a telescope, if necessary, so that he may watch a pendulum, or any other object having a periodic motion, in sight of both observers. Either the length of the pendulum, or the distance between the observers is then varied until a sharp sound made by A, when the pendulum is at the middle point of its swing, is heard by B at the moment when the pendulum, after completing one or more oscillations, again passes the middle point. The distance is then measured, and the time of the pendulum determined. Measurements must also be taken in which sounds made by B are heard by A as the pendulum passes its middle point. The experiment is then repeated with a distance between the observers (¶ 135, 3) two or three times as great as before.

Other methods of measuring short intervals of time will be considered in experiments which follow.

¶ 138. **Proper Methods of Counting.** — In counting the ticks of a watch (which usually occur at intervals of one-fifth of a second), it will be found difficult, if not impossible, to repeat, even mentally, the names of numbers which contain more than one syllable.[1] In the following method of counting, this difficulty is avoided: —

---

[1] See Ex. 30, Elementary Physical Experiments published by Harvard University.

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	**2**
1	2	3	4	5	6	7	8	9	**3**

By counting the ticks which actually occur *within* a given interval of time, the length of that interval will on the whole be fairly estimated. There is, however, a tendency in most persons to count one too many ticks. When a given interval contains a whole number of ticks, one occurring at the beginning of the interval should be counted "nought," or not counted at all. Obviously the first and last tick should not both be counted.

With intervals of time (as with intervals of space), care must be taken to distinguish the number of intervals from the number of divisions between which they lie. In the same way that the zero of a scale should not be counted "one," the beginning of an interval of time should not be called one second or one-fifth of a second. A miscount may generally be avoided by pronouncing the word "now" at the beginning of the interval, then beginning the count immediately afterward.

An accurate method of counting is important in a great variety of measurements, especially those which involve rates of vibration or revolution. The student should consider carefully what habits he has formed in this respect, and if they are not good, whether it is preferable to change them, or to make an allowance for "personal error" in each separate determination.

---

[1] The difficulty is greatly lessened by counting every other tick; but on account of the greater inaccuracy, this method of counting is not generally recommended.

EXPERIMENT LII.

GRAPHICAL METHOD.

¶ 139. **Determination of Rates of Vibration by the Graphical Method.**[1] — A tuning-fork (*ae*, Fig. 125) making from 100 to 300 vibrations per second, and a pendulum (*bf*), made of an ounce bullet (*f*) and a piece of clock-spring (*b*), are mounted as in the figure, so that when the tuning-fork and pendulum are in vibration, two short and fine brass wires attached one to each may make marks (*h* and *i*, Fig. 126) as close together as possible on a piece of smoked glass.

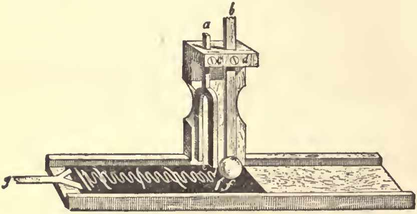

Fig. 125.

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[1] The experiment here described is essentially the same as that given in Exercise 31, Elementary Physical Experiments, Harvard University. This application of the graphical method is due to Prof. Hall.

The tuning-fork and the spring are then firmly clamped by the screws *c* and *d*.

The smoked glass is now drawn slowly out from under the pendulum and the tuning-fork. The points of the wires *e* and *f* should draw a single line (*h i x*, Fig. 126) upon the surface of the glass. If they do not, the wires should be bent, or their relative position otherwise adjusted. The smoked glass is now to be replaced, and both the pendulum and the tuning-fork are to be set in vibration, — the latter by drawing a violin-bow across one of the prongs. The bow must be drawn slowly at first, and always *in a direction nearly parallel to the vibration* which it is desired to create. That is, the bow should be held at right-angles to the prongs, but nearly parallel to the plane containing them. The smoked glass is again drawn out from under the pendulum and the fork, with a slow but uniform velocity.

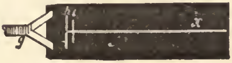

FIG. 126.

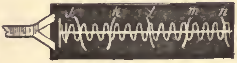

FIG. 127.

The wire attached to the tuning-fork, partaking of its vibration, will trace upon the glass a series of waves. The wire attached to the pendulum would similarly trace a series of much longer waves, were it not that owing to the amplitude of its oscillation, the wire usually leaves the glass at the extreme points of a swing. The result is a series of marks (*j*, *k*, *l*, etc., Fig. 127).

The time required for one complete oscillation of the pendulum is represented by the distance between alternate marks (*j* and *l*, or *k* and *m*, Fig. 127). The number of complete vibrations made by the tuning-fork in the same length of time is to be found by counting the waves executed in the same distance. Thus between *j* and *l* there are (in the figure) about 6¼ complete, or 12½ half-waves; and between *l* and *n* there are similarly about 7 waves. In practice, a much greater number would be counted.

If the waves are perceptibly closer together at *k* or at *l* than at *m* or at *n* (or the reverse), the glass has not been drawn with sufficiently uniform velocity. In this case, instead of depending upon the marks (*j*, *k*, *l*, etc.) actually made by the pendulum, it is necessary to draw a line at a distance from each mark equal to that between *h* and *i* (Fig. 126), and at the left or at the right of it, according to whether *h* is at the left or at the right of *i*. The new lines show where the wire attached to the pendulum *would have crossed* the glass, provided that it could have been made absolutely coincident with the wire attached to the pendulum. By the use of lines drawn as above, we may in counting the waves avoid errors due to irregularity in the speed of the glass. The number of whole waves included between two alternate lines should be recorded in each case, together with an estimate of the fractions of a wave left over at each end of the series. This fraction should be expressed in tenths § (26).

To find the rate of vibration of the tuning-fork, the time occupied by one complete oscillation of the pendulum must now be determined. This is done by timing, let us say, one hundred complete oscillations. Having given a signal, one observer begins to count the oscillations of the pendulum, while a second observer, as soon as the signal is perceived, begins to count the ticks of a watch (see ¶ 138). When the pendulum has completed a given number of oscillations, the first observer signals to the second to stop counting.

The number of complete oscillations of the pendulum per second is found from the time required for 100 or 200 oscillations (as the case may be), by simple division, and the result is multiplied by the average number of waves made by the fork during one of these complete oscillations to find the "vibration number," or "pitch" of the fork, — that is, the number of complete vibrations made in one second.

EXPERIMENT LIII.

BEATS.

¶ 140. **Theory of Beats.** — When two musical notes, nearly but not quite in unison, are sounded together with about the same degree of loudness, the effect upon the ear is by no means uniform. At regular intervals the sound swells out, and these intervals are separated by moments of comparative silence. Each rise and fall of the sound constitutes a "*beat.*"

The increase is due to the mutual re-enforcement of the two sets of vibrations communicated to the air; the decrease is caused by the interference of these vibrations.

Let us suppose that two tuning-forks, one making 256, the other 255 vibrations per second, are started at a given instant by forcing their prongs together and suddenly releasing them. The prongs of both forks will spring apart simultaneously, and each fork will cause a slight condensation of the air on each side of it. This condensation will be followed by a rarefaction when the prongs rebound, then by several alternate condensations and rarefactions, nearly though not quite synchronously performed. The result is that the vibrations reaching the ear at the same distance from both forks are very much greater than if one fork were sounding alone. At the end of half a second, however, the first fork will have made 256 ÷ 2, or 128, complete vibrations; so that, as at the start, its prongs will be springing apart; but the second fork will have made only 255 ÷ 2 or 127½ vibrations, so that its prongs will be approaching each other. The condensation produced by one fork will tend to offset the rarefaction produced by the other. The effect on the ear will accordingly be less than if one of the forks were sounding alone. This interference of the vibrations will evidently continue as long as the forks are vibrating in opposite ways. At the end of a second, the first fork will have made just 256, the second fork just 255 complete vibrations, and the direction in which the prongs are moving will be in each case the same as at the start, and hence the same for both forks. The sounds will therefore re-enforce each other as at first. It is evident that, with the forks in question, periods of re-enforcement must occur every second, separated by intervals of interference. In other words, two forks making 256 and 255 vibrations per second must give rise to 1 "beat" per second when sounded together.

In the same way it may be shown that two forks differing by n vibrations per second give rise to n beats per second. In other words, when two musical notes are nearly in unison, *the number of beats per second is equal to the difference between the vibration numbers* corresponding to the two notes in question.

¶ 141. **Determinations of Pitch by the Method of Beats.** — The special apparatus required for this experiment consists of a series of tuning-forks with differences of from three to five vibrations per second, covering an interval of one octave (¶ 134). The first and the last of the series are to be sounded together, to make sure that the musical interval is exact. If the forks are nearly but not quite an octave apart, faint beats may be heard. In this case one of the forks must be loaded with small bits of wax near the end of its prongs until the beats disappear. If the wrong fork is loaded the beats will become more frequent than before. The same effect may be produced if *too much* weight is added to *either fork;* hence care must be taken at first to add very little weight at one time.

The simplest way in general to tell whether a fork is higher or lower than may be required for the purposes of harmony is by the method of loading suggested above. The effect of the additional weight is to lower the rate of vibration of the fork to which it is attached. Whenever by loading a fork it may be brought into harmony with a given musical note, we know that fork to have a higher rate of vibration than the purposes of harmony require.

If, for instance, the first fork in the series gives 61, and the last 120 vibrations per second, the first will have to be loaded until it gives 60 vibrations per second, in order to be in harmony with the other fork. Again, if the second fork gives 64 vibrations per second, it will have to be loaded to bring it in unison with the first fork. We may generally assume that the forks are arranged by the instrument-maker in an ascending series.

The experiment consists in a determination of the number of beats produced in a given length of time by sounding together each pair of consecutive forks in the series, that is, the first and second, the second and third, the third and fourth, etc. The student will do well to begin counting with one of the beats which happens to occur when the second-hand of his watch indicates a round number. The *beginning* of this beat should not be counted (see ¶ 138). One hundred beats should be timed if possible. The time of the last beat should be observed to a fraction of a second. The number of beats per second should be calculated in each case.

The results represent differences between each pair of consecutive forks in the series; hence when added together we have the difference between the first and the last in the series, for the whole difference in question must be equal to the sum of all its parts.

Now two notes an octave apart are to each other, in respect to their vibration numbers, as 2 is to 1 (¶ 134); hence the last number in the series is twice the first. It follows that the difference between the first and last numbers is equal to the first number in the series. The result of adding together the numbers of beats per second is therefore to find the number of vibrations executed by the first fork in one second.

By adding to this number the number of beats per second between the first fork and the second fork we find the pitch of the second fork; and in the same way, successively, the pitch of each fork in the series can be calculated.

EXPERIMENT LIV.

LISSAJOUS' CURVES.

¶ 142. **Theory of Lissajous' Curves.** — We have seen, in Experiment 52, that when a piece of smoked glass is drawn beneath a pointed wire attached to a vibrating tuning-fork, a wave-line is traced upon it. If instead of drawing the glass completely away from the tracer, the motion be suddenly reversed, we shall evidently obtain a double wave which will resemble one of the figures below (Fig. 128, 1, 2, and 3) according to the point (*a*) in the curve at which the reversal takes place. In the first curve the two waves happen very nearly to coincide. We may imagine the reversal to take place so that there should be a perfect coincidence.

Now let us suppose that when the tracer reaches a certain point, *b*, a second reversal takes place, and a third reversal occurs when the tracer returns to the former point, *a*. Evidently, if the reversals are properly timed, the tracer will follow the same path over and over.

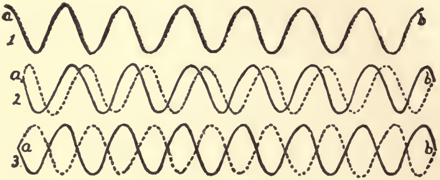

Fig. 128.

In practice we obtain a similar result by attaching a small piece of smoked glass to the larger of two tuning-forks. When the larger fork makes one vibration in the same time that the smaller fork makes for instance 8, we obtain tracings as in Fig. 129, 1, 2, or 3, according to the relation which happens to exist between the forks at the start.

These are examples of Lissajous' curves. The reversal of the smoked glass is not sudden, as in the case previously supposed, and its velocity is greatest when the middle of the figure is being drawn. This accounts for the difference in appearance between these curves and those represented in Fig. 128.

It may be shown that whenever two vibrations at right-angles are compounded graphically, as in Fig. 129, unless the times of the vibrations are incommensurate, a Lissajous' curve results. Each musical interval (¶ 134) has, accordingly, its characteristic curves. These curves are in general too complicated to be discussed in an elementary work. We shall confine ourselves to such cases as are represented in Fig. 129, where one fork makes a certain whole number of vibrations while the other makes one.

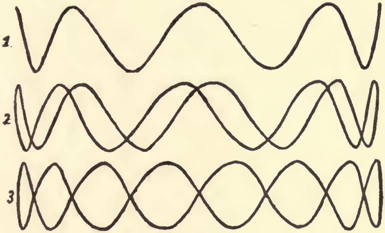

FIG. 129.

To find in such cases the musical interval between the forks, we have to experiment until a figure like the third is obtained (Fig. 129, 3). If this figure contains *n* lobes, then the higher fork makes *n* times as many vibrations as the lower fork.

It has been so far assumed that the two forks are separated by an exact musical interval, so that at the end of a certain period they find themselves in exactly the same mutual relation as at the start. If this is not the case, it is evident that the tracer will not follow the same path in all cases, but that this path will be continually changing.

Let us suppose that the tracer reaches its highest point, as seen in the figure, when the glass reaches its extreme right-hand or left-hand turning-point. Then the curve traced will be represented as in Fig. 129, 1. If the small fork is a little behind-hand we shall have a tracing as in Fig. 129, 2; and if the small fork has only reached the middle of its course when the glass turns, we shall have a tracing like Fig. 129, 3. Evidently, if the small fork starts as in (1) and falls slowly behind the other, we shall have a series of tracings represented by (1), (2), and (3). It is not until the higher fork has fallen one complete vibration behindhand that the same figure will be repeated.

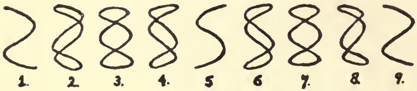

Fig. 130.

If the smaller fork is gaining instead of losing, a similar series of changes will be produced. There is in fact no way to tell which fork is too high for the musical interval in question, except as in the last experiment, by loading it and observing the result. A complete cycle of changes in the case of two forks one octave and one fifth apart (¶ 134) is shown in Fig. 130.

The symmetrical lobed figures (3 and 7) appear twice in a cycle; the serpentines appear also twice; but one of them is left-handed (1), the other right-handed (5). The interval between two left-handed (or that between two right-handed) serpentines always represents one complete cycle, and is accordingly equal to the time in which the higher fork makes one whole vibration more or less than would be required to give a perfect musical interval.

Let p be the pitch of the lower fork, that is, the number of vibrations it makes in one second, and let n denote the approximate musical interval between the forks; then the pitch of the higher fork, which we will call P, must be equal to np, nearly. If, however, we observe c cycles per second, the true pitch of the higher fork is $np \pm c$. Here c is positive if by loading the higher fork the musical interval may be made perfect; if on the other hand the lower fork must be loaded, c will be negative. With this understanding we have

$$P = np + c. \qquad \text{I.}$$

and

$$p = \frac{P - c}{n}. \qquad \text{II.}$$

These formulæ apply only to cases in which, as we have supposed, n is a whole number.

¶ 143. **Determination of Pitch by Lissajous' Curves.** — A tuning-fork of known pitch (Exps. 52 and 53) and one approximately an octave above or below it are to be mounted, as in Fig. 131, with their prongs at right-angles. The prongs of one fork (A) are to be coated with lampblack, except at a small point where, by the touch of a pin, the bright metallic surface is made visible. Opposite this point on the other fork (B) a lens, C, of about 1 inch focus, is to be attached with sealing-wax, at such a distance that a highly magnified image of the point may be seen

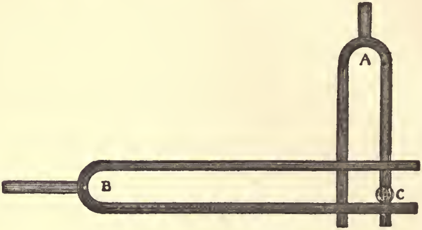

Fig. 131.

through the lens. When a violin-bow[1] is drawn across the fork A, the bright spot partaking of the vibration will be apparently extended into a horizontal line, Fig. 132.

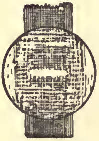

Fig. 132.

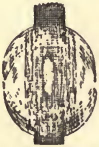

Fig. 133.

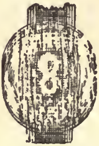

Fig. 134.

When the fork B is set in vibration, the motion of the lens will cause the spot to be apparently elongated into a vertical line, as in Fig. 133. When both forks vibrate simultaneously the vertical and horizontal motions will be combined, and if the forks are separated by an exact octave, one of Lissajous' curves will be formed, as for instance in Fig. 134.

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[1] In practice, it will be found convenient that one or both of the forks should be maintained in vibration by electrical means.

If this curve is permanent in form, the experiment is now finished; but if, as is generally the case, it passes through a series of cycles, as in Fig. 130, ¶ 142, it becomes necessary to count the number of complete cycles which take place in a given length of time. It is also necessary to load one of the forks, as in ¶ 141, until the changes in the cycles become less frequent.[1]

We thus find whether *c* is positive or negative in the formulæ of ¶ 142. The pitch of one of the forks is finally to be calculated by one of the formulæ in question from the pitch of the other fork, previously determined.

EXPERIMENT LV.

THE TOOTHED WHEEL.

¶ 144. **Construction of a Toothed-Wheel Apparatus.** — A toothed-wheel apparatus capable of giving fairly accurate results is represented in Fig. 135, as seen from above. A vertical cross-section is shown also in Fig. 136. The works (*e*) of an ordinary eight-day spring clock, from which the escapement has been removed, are mounted on a piece of wood, and a disc of cardboard (*a*) is attached to the axle usually carrying the second hand. Two pieces of watch-spring are also attached to this axle at *b*, and bent into loops so that two small loads (*c* and *d*) which they bear may hang quite close together when the wheel is at rest. The friction which the springs exert against the air acts as a governor upon the speed of the machine. The velocity of rotation will be found to vary very little as the force of the main-spring grows less and less. To make the wheel turn faster, the loads (*c* and *d*) may be decreased; or a *slight* change may be produced by winding up the main-spring. To make the wheel go slowly, the load may be increased; or a slight decrease in speed may be had either by waiting for the main-spring to unwind itself, or by applying friction to one of the more slowly moving wheels. The upper surface of the disc, *a*, should be painted black. The number of revolutions which it makes in a given time may be counted by watching a white spot upon it, or still better by listening to the sound

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[1] It is possible to load a fork so that a figure of a certain class (see Fig. 130, 1–9) may preserve its characteristics until the vibration dies away.

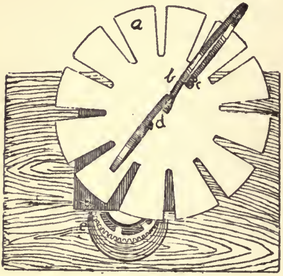

Fig. 135.

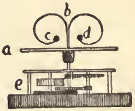

Fig. 136.

made by an object striking lightly against a projection from the wheel or from the axle upon which it is mounted. At equal distances around the circumference of the wheel, narrow radial slits should be cut out. The number of slits must be made with reference to the usual speed of the machine and the number of vibrations per second which the toothed wheel is intended to measure. The wheel represented in Fig. 135 makes about 8 revolutions per second without any load, — the speed being reduced to 4 revolutions per second by a load of a few grams at *c* and *d*. With twelve notches in the disc, this apparatus affords from 48 to 96 nearly instantaneous views of objects seen through the rim of the wheel. The instrument is accordingly suited to the determination of the pitch of tuning-forks making from 48 to 96 vibrations per second. It may also be used for much higher forks, as will be presently explained.

¶ 145. **Theory of the Toothed Wheel.** — By the apparatus just described we are able to obtain at regular intervals a series of instantaneous views of a vibrating object. If the intervals between the views correspond to the period of vibration in question, the same view will evidently repeat itself over and over. If the intervals are sufficiently short, the effect will be a continuous impression upon the eye. Thus when the eye is held close behind the rim of the rotating disc (Fig. 135), the speed of which is properly adjusted, we may obtain a series of views of a tuning-fork, in all of which the prongs are, for instance, at their greatest elongation. The result is that the fork appears to be at rest. To obtain this result the number of slits which pass in front of the eye in one second must be equal to the number of vibrations executed by the fork in the same time. If the wheel is moving a little too fast or too slow, the successive views of the fork will not be exactly the same.

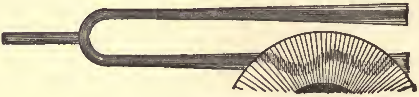

Fig. 137

The position of the prongs will seem to change as if the fork were executing a very slow vibration. When the fork is held close behind the rim of the disc, as in Fig. 137, a different effect is produced.

Let us first consider the effect of a single slit moving along the fork. Let 1, 2, 3, 4, 5, 6, 7, 8, Fig. 138,

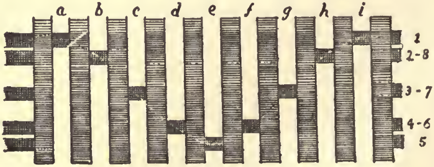

Fig. 138.

be views of the fork seen through such a slit when occupying the successive positions *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i*. These views are evidently situated along the dotted line *ai*. Let us now supply the intermediate views. We shall evidently have the curve shown in

Fig. 137, or in *ab*, Fig. 139. Now let another slit pass along the fork. We shall have similarly a curve, *cd* or *ef* (Fig. 139), which may or may not coincide with *ab*. If it does not coincide with *ab*, we shall probably not see either of the curves, since the light reflected through the slits will hardly have time to affect the eye. If, however, several such curves coincide, the joint effect will be similar to that shown in Fig. 137.

In order that successive curves may coincide, it is necessary that successive slits should reach a given point in the curve (as *a*, Fig. 138) at the same instant that the prong of the tuning-fork reaches that point.

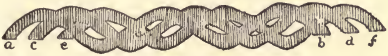

Fig. 139.

In other words, the interval of time between the arrivals of successive slits must correspond with the period of the tuning-fork.

It will be found, if a toothed wheel is adjusted so as to show waves, as in Fig. 137, that when the speed is increased the waves will seem to follow the direction in which the wheel is moving. while if the speed is lessened, the waves will move in the opposite direction. This is the result of a series of wave images (see Fig. 139), each of which is situated in a *slightly* different place from the one preceding it. The direction in which the waves seem to move is a valuable guide in adjusting the speed of the wheel.

It is easy to trace out in a similar manner the appearance of a vibrating fork for any speed of the wheel. Usually it will appear blurred, as if looked at in the ordinary manner. If, however, the wheel is moving twice as fast as it ought, a double wave will be visible, as in Fig. 140. If, again, the fork makes in one second a number of vibrations twice as great as the number of slits which pass a given point, the appearance of the fork will be as in Fig. 141. Care must be taken not to mistake this curve for the double curve of Fig. 140, nor for the regular curve of Fig. 137. We notice that in Fig. 141 there are two complete waves in the distance between two successive slits (*a* and *b*).

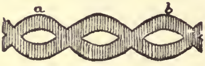

Fig. 140.

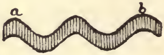

Fig. 141.

In the same way this distance will be divided into *n* waves if the fork executes *n* vibrations between successive views from a given point.

By this principle we may find the rate of a fork too high to be measured by the ordinary method.

¶ 146. **Determination of Pitch by means of a Toothed Wheel.** — The experiment consists simply in adjusting the speed of a toothed wheel (Fig. 135, ¶ 144) so that a fork held behind the rim of a wheel (as in Fig. 137, ¶ 145), and making about 64 vibrations per second, will be apparently thrown into simple stationary waves, the lengths of which will be equal to the distance between the teeth of the wheel, then finding how many teeth pass by a given point in one second. We have already considered (¶ 144) the manner in which the speed of the wheel may be adjusted and how the number of revolutions may be counted.[1] The number of revolutions made in one second multiplied by the number of teeth gives the number of teeth per second. This is (see ¶ 139) the "pitch" of the tuning-fork.

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[1] If it is found impossible to adjust the speed exactly, or to keep it adjusted, accurate results may still be obtained by counting the number of waves which in one second traverse the field of view. This number is to be added to the number of slits passing a given point in one second if the motion of the waves is opposite to that of the wheel; if both move in the same direction the first number is to be subtracted from the second.

DYNAMICS.

¶ 147. **Different Methods of Measuring Velocity in Dynamics.**—When a body is moving so slowly that it is possible to make a series of observations of its position at different points of time, no particular difficulty is met in the measurement of its velocity. Thus in Exp. 60, to find the average velocity of a ring rotating about its axis, we observe the distance traversed between two ticks of a clock, and divide it by the interval of time in question. Such slow motions are, however, the exception in dynamics. In certain cases instantaneous photography has been employed for the study of rapid motions. The estimation of velocity generally requires, however, special devices, such as have been employed for the velocity of sound (Exp. 51).

(1) In rough measurements, we frequently make use of the sounds produced by a moving body when it strikes different obstacles in its course. A familiar example of this method consists in the determination of the speed of a railway train by counting the number of rails crossed in a given length of time. To find the velocity of a marble rolling in a groove, small tacks may be driven into the groove at such distances that the successive sounds made by the marble in crossing them correspond with the ticks of a clock. The regular increase of velocity caused by a steady

incline is then easily demonstrated by measuring the distances between the tacks.

(2) By substituting for a series of tacks a series of electrical connections which are made or broken by a moving body, we may make use of any of the devices by which time is measured by electrical agency.[1]

The velocity of a rifle bullet has been measured by the interval of time between the rupture of two wires a known distance apart. The time of rupture is usually recorded "*graphically*" by means of a chronograph (see ¶ 266). Curves traced simultaneously by the armature of an electrical sounder and by a tuning-fork (see Exp. 52), enable us to estimate precisely exceedingly small intervals of time.

(3) There are various devices in which the motion of a body may be directly recorded by the graphical method. Thus, in Morin's Apparatus (Fig. 142), a pencil (*c*) attached to a falling body marks directly upon a revolving cylinder covered with paper. If the rate

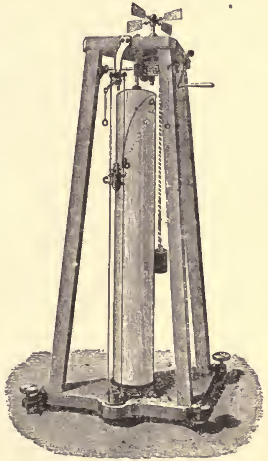

FIG. 142.

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[1] See Trowbridge's New Physics, Exp. 71, 72, 73.

of revolution is known, we may obviously infer the position of the body at different points of time from the tracing (*ab*) made by the pencil.

Another device in which the vibrations of a tuning-fork attached to a falling body may be made to indicate its position, will be found in Trowbridge's New Physics, Exp. 74.

A simple instrument illustrating the graphical method of measuring velocity will be described in the next section.

(4) In studying the motion of fluid streams, the velocity is frequently calculated from the size of a tube or orifice, and from the volume which flows through this tube or orifice in a given time. Thus if a stream of water issues from an orifice $\frac{1}{4}$ *sq. cm.* in cross-section at the rate of 25 *cu. cm. per sec.*, its velocity at the orifice must be 100 *cm. per sec.* This principle has been applied to illustrate the law of falling bodies. A stream of water projected horizontally with a known velocity must traverse a known horizontal distance (*BC*, Fig. 143) in a known time; hence the time required for gravity to deflect the stream through a known vertical distance (*AB*) is determined.

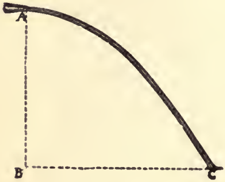

FIG. 143.

FIG. 144.

(5) The pressure of a stream of gas has been applied to the determination of the mass of the gas when its velocity is known, and conversely for a determination of its velocity when the mass is known. If, for instance, a mass of gas m, impinging with the velocity v, on a scale-pan (a, Fig. 144) causes a force, f, to be exerted for a time t, we have from the general formula (§ 106)

$$m = \frac{ft}{v}; \quad v = \frac{ft}{m}.$$

(6) The laws of falling bodies are frequently made use of for indirect measurements of velocity. Thus since a body is known to fall 4.9 metres in 1 second, the velocity of a stream of water projected horizontally at a distance of 4.9 metres above a certain level will be equal numerically to the horizontal distance traversed before reaching that level, the time in question being 1 second. Again, the velocity of a pendulum when it passes its central point may be estimated by the distance it has fallen in reaching that point, or by the distance it rises after reaching that point (see § 109).

(7) The law of *action and reaction* enables us to make comparisons of velocity. Thus if a bullet of mass m, striking a log of mass M, suspended as in Fig. 145, gives it a velocity V (see § 106), the velocity of the bullet (v) may be found by the equation,

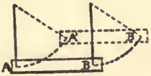

FIG. 145.

$$v = \frac{(m + M)}{m} V.$$

Changes in velocity may be measured by the same principle. If two billiard balls, *A* and *B* (Fig. 146), are suspended by cords of equal length so as to just touch each other without pressure, and if the greater, *A*, is drawn aside to a position A' (Fig. 147) and allowed to strike *B* while resting at B', the latter will reach a position B'', while the former reaches A''. The velocity acquired by *A* in falling from A' to *A* will be proportional to the straight line $A'A$ (§ 109); the velocity after impact will be proportional to AA'' and in the same direction as before; hence the loss will be proportional to $A'A - AA''$. At the same time *B* gains a velocity represented by $B'B''$.

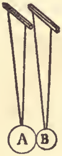

Fig. 146.

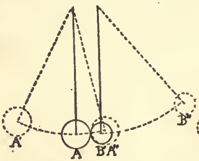

Fig. 147.

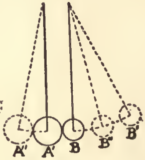

Fig. 148.

If on the other hand *B* strikes *A* from a position B' (Fig. 148), it will rebound to B'' in the opposite direction; hence its change of velocity will be $B'B + B''B$. The corresponding gain of velocity by *A* will be represented by $A'A''$.

It is easy to show by experiment that the products of the masses and their respective changes of velocity are equal, whether the balls are elastic or inelastic.[1]

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[1] See Ex. 20 of the Descriptive List of Elementary Physical Experiments published by Harvard University.

A comparison of the changes of velocity in question gives a simple means of estimating the relative masses of the balls.

EXPERIMENT LVI.

FALLING BODIES.

¶ 148. **Determination of Distances traversed by Falling Bodies in Different Lengths of Time.** — A wooden rod, *jp* (seen edgewise in Fig. 149), about 25 *cm.* in length, 3 *cm.* in breadth, and 1 *cm.* in thickness, is suspended from the edge, *f*, of a bracket, *ef*, by a strap of paper forked at *h*, so that the rod, when free, may hang in a vertical position. An ounce bullet is next suspended by a thread from the peg, *c*, and lowered to a position, *q*, near the bottom of the rod. The bracket is then moved (by loosening the screws *d* and *g*) so that the rod may barely touch the bullet. Then the bullet is removed, and either the rod is smoked at *j* and at *p*, or pieces of smoked paper are attached to it at these points.

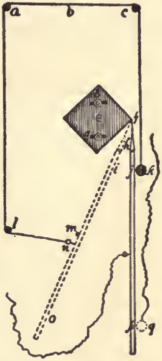

FIG. 149.

The bullet is now suspended at a point, *k*, near the top of the rod, by a thread passing over the smooth round pegs *c*, *a*, and *l*, to a screw-eye, *n*, near the middle of the rod. The rod is drawn one side by the pull on the thread, due to the weight of the bullet. Care must be taken to ease the thread round the pegs, so that the true position of equilibrium may be found. A pin *m* may then be placed so as to mark this position of equilibrium.

To find the height of the bullet a finger is laid upon the thread at *a*, and the thread is slipped off the peg *l*, so that the rod may strike the bullet. A mark will thus be made on the smoked surface at *j*. The thread is now carefully replaced on the peg *l*, so that the tension may be the same as before. When the finger is finally removed from *a*, there should be no slipping of the thread. If there is, the experiment must be repeated, until the bullet, having made a mark on the rod, remains unchanged in position.

Any oscillation of the bullet must now be arrested by lightly pushing the thread, just below *c*, in a direction always opposite to that in which the bullet is swinging, or simply by allowing time enough for it to come to rest. The thread is then burned at *b* by holding a lighted match under it. The rod and the bullet will thus be released at very nearly the same instant. When the rod reaches its vertical position, *jp*, it will strike the bullet at some point, *q*, where the bullet will make a mark on the smoked surface.

The distance between the two marks, one near *j*, the other near *p*, is now to be measured. This distance is equal to that through which the bullet falls while the rod is reaching its vertical position ; that is, in half the time it takes the rod to swing from one side to the other. To determine the time in question, we set the rod once more in oscillation and find how long it takes it to complete 100 or more swings.[1]

To obtain the best results, the oscillations should be timed as will be explained in the next experiment. The time of a single oscillation (either from left to right or from right to left) is then calculated and divided by 2, to find the time occupied by the rod in reaching its vertical position in the middle of one swing. This gives the time occupied by the bullet in falling through the observed distance.

The experiment should be repeated with the same apparatus until results are obtained agreeing within 2 or 3 per cent. The experiment should be then varied by using rods of different lengths. The results should be entered as follows: in the first column, the distance through which the bullet falls; in a second column, the corresponding times of falling; in a third column, the squares of these times, in a fourth column, the ratios of the distances to the squares of the times. Thus: —

1. Distance Fallen.	2. Time Occupied.	3. Square of Time.	4. Ratio of 1 to 3.
19.2 *cm.*	0.20 sec.	0 040	480
80.0	0.40	0.160	500
etc.	etc.	etc.	etc.

It will be seen by the formula $d = \frac{1}{2} gt^2$ (§ 108) that the ratio of the distance to the square of the time must be equal to $\frac{1}{2} g$, which is the distance a body falls in one second. The numbers in the fourth column may be considered, therefore, as different estimates of this distance, founded on observations lasting through different intervals of time. These estimates should evidently show an approximate agreement; but the results are modified somewhat by the fact that we are not experimenting with a body which is perfectly free to fall. A device, similar in many respects to that shown in Fig. 149, will be found described in Exp. 20 of the Descriptive List of Experiments in Physics, published July, 1888, by Harvard University. A device in which two electromagnets are used to set free a pendulum and a falling body will be found in Trowbridge's New Physics, Exp. 67.

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[1] The student should notice that though the swings grow shorter and shorter in length, there is little or no perceptible change in the rate of oscillation (see § 111). A more exact method of testing this point will be met incidentally in Exp. 58.

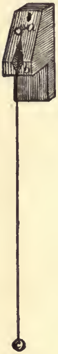

Fig. 150.

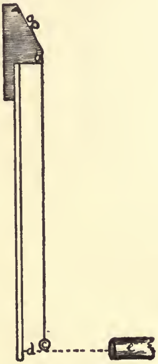

Fig. 151.

EXPERIMENT LVII.

LAW OF PENDULUM.

¶ 149. **Determination of Times of Oscillation.** — An ounce bullet (*c*, Fig. 150) is to be suspended by a waxed silk thread, passing through a notch (*b*) in the edge of a bracket to and round a pin, *a*, by which the thread can be lengthened or shortened. The lower surface of the bracket must be horizontal (see *b*, Fig. 151), and the groove must be deep enough to reach this surface. It is now required to find the length of the pendulum thus constructed; that is, the distance from its point of suspension, in the surface, *b*, to the middle of the bullet, *c*. This is done by means of a wooden rod, *bd*, graduated in millimetres. The rod is held parallel to the thread (and hence vertical) with its zero at *b*. The height of the centre of the bullet is found from that of the top and bottom by taking the mean. To avoid parallax (§ 25) these heights are sighted through a telescope (*e*), on the same level with them. We thus find the length of the pendulum in question. The time occupied by a hundred or more consecutive[1] oscillations of the pen-

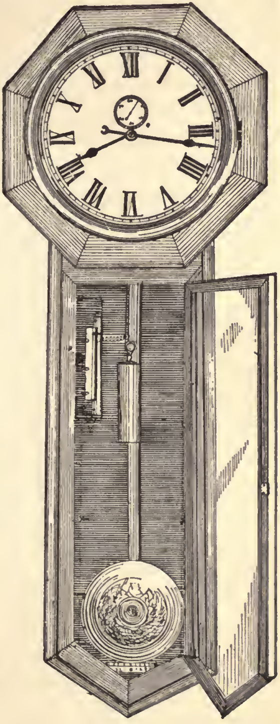

FIG. 152.

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[1] The importance of observing long series of consecutive observations must not be overlooked. A student is apt to imagine that 10

dulum is now to be found. The counting is to be begun at a moment when the pendulum begins a swing just as the second hand of a regulator (Fig. 152) indicates some round number of seconds. The time must be written down provisionally in advance. Let us suppose that the observer is ready at 11h. 8m. He writes down provisionally 11h. 9m. 0s. When at a given tick of the clock the second-hand indicates 40 sec. he counts that tick 40, the next 41, etc.[1] If on the 60th tick the simple pendulum does not happen to be at the beginning of a swing, a new trial should be made (e. g.) at 11 h. 10 m. 0 sec. In the course of about 10 trials a case should be found in which a swing *seems* to begin exactly on the minute and second provisionally recorded. The observer then begins immediately to count the swings completed by the pendulum (¶ 138). When the pendulum has made, let us say, 100 swings, the time by the clock is again noted. If the clock ticks just as the pendulum completes its 100th swing, the indication of the clock (which will not change for one second) is exact; if, however, the 100th swing is completed half-way between two ticks of the clock, the first indication should be increased by 0.5 seconds. The student should practise in this way the estimation of half-seconds or smaller fractions if possible. The results are invariably to be expressed in tenths.

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series of 10 swings each gives an average result as good as one series of 100 swings, whereas in fact, 100 series of 10 each would be required (see § 51).

[1] With a little practice, the student should be able to follow the motion of the second-hand for some time by simply counting ticks, *without looking at the clock.*

The experiment is to be repeated with pendula, the lengths of which are about 10, 20, etc., up to 100 *cm.* The results are to be arranged in two columns, the first showing the length of the pendulum, or the distance from the point of suspension to the centre of the bullet; the second showing the time of vibration, found by pointing off two decimal places from the time in seconds occupied by 100 vibrations. Then a third column is to be calculated, showing the squares of the times of vibration; and a fourth column showing in each case the ratio of the length of the pendulum to the square of the time of vibration. Thus: —

(1) Length of Pendulum.	(2) Time of Swing	(3) Square of Time	(4) Ratio of (1) to (3
8.8 *cm.*	0.30 sec.	0.09	97.8
99.0	1.00	1.00	99.0
etc.	etc.	etc.	etc.

In accordance with the well-known law of the pendulum (§ 110), the squares of the times in column (3) should be proportional to the lengths in column (1); hence the numbers in the fourth column should be (theoretically) the same. In practice variations occur, due not only to errors of observation, but also to the fact that a bullet suspended in air by a silk thread is only an approximation to an ideal simple pendulum.[1]

By comparing the table found in this experiment with that obtained for falling bodies in Exp. 56, we discover a curious relation. The length of a pendulum which makes one swing in one second is about 99 cm. The distance a body falls in one second is about 490 cm. The latter is nearly 5 times as great as the former. Again, the length of a half-second pendulum is not quite 25 cm., the distance a body falls in half a second is about 122 cm., that is, nearly 5 times as great as the corresponding length of the pendulum. This proportion will be found to exist in every case.

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[1] A pendulum consisting of a small sphere suspended by a fine thread is sometimes called a simple pendulum. An ideal simple pendulum consists, however, of an infinitely small body suspended in vacuo by a perfectly flexible but inextensible cord entirely devoid of weight. See Deschanel's Natural Philosophy, Chapter VI.

It is obvious that if this proportion is known,[1] we may calculate the distance through which a body falls in a given time from the length of a pendulum making one swing in the same time. We shall make use of this principle in the next experiment.

EXPERIMENT LVIII.

METHOD OF COINCIDENCES.

¶ 150. **Adjustment of a Pendulum of Peculiar Construction.** — A serviceable device, which conforms approximately to the conditions required of a simple pendulum, is represented in Fig. 153 as seen from in front, and in Fig. 154, in profile. It consists of a cylinder (gj) suspended by two vertical loops of silk thread passing around the horizontal pins *ab* and *hi*. The diameter of these pins should be exactly the same, and not over 1 *cm*. Their length should be about 10 *cm*. The upper pin (*ab*) is driven through a fixed support; the lower pin should pass as nearly as possible through the centre of gravity of the cylinder. The ends of the thread, after passing over the pin *ab*, are carried each to one of the pins *c*, *d*, *e*, and *f*, by turning which the threads may be lengthened or shortened. A disc is also attached to the cylinder, and in this disc are made two *V* shaped holes (*g* and *j*). Opposite the lower hole (*j*) may be placed an opening (*k*), in a shield, through which instantaneous views of objects behind the pendulum may be obtained at regular intervals. A small wire loop may be attached to the pendulum so as to complete an electrical connection between two drops of mercury at *l* when the pendu-

---

[1] The law of falling bodies gives (§ 108) $d = \frac{1}{2} gt^2$; the theory of the pendulum gives (see Appendix) $l = \frac{gt^2}{\pi^2}$; hence we have $d : l :: \pi^2 : 2 = 4.935 : 1$, nearly. This ratio is not affected by the value of g, but is slightly affected by the resistance of the air.

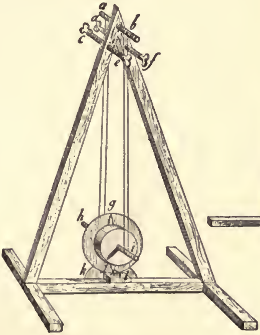

FIG. 153.

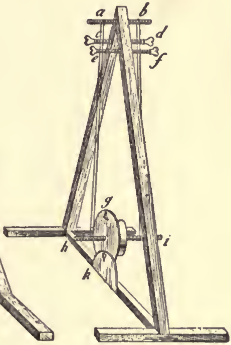

FIG. 154.

lum is at rest or in the middle of a swing. The length of the pendulum thus constructed is found by measuring the distance between these pins from centre to centre. In the absence of a cathetometer (¶ 262) or other device by which the distance in question may be accurately measured, it is well to adjust it by turning the pins *c*, *d*, *e*, and *f* until a metre rod fits without looseness or pressure between the pins *ab* and *hi*, so as to subtend the vertical distances either between *a* and *h* or between *b* and *i*. The diameters of the pins at *a*, *b*, *h* and *i* are now measured by a vernier gauge (Part I. ¶ 50). The average diameter added to the length of the metre rod gives the distance between the pins from centre to centre.

In regard to the working of this pendulum, it may be pointed out that the cords (*ah* and *bi*) keep the pins (*ab* and *hi*) parallel, hence horizontal, and always the same distance apart. The centre of the pin *hi* swings, therefore, in a vertical plane about the middle point of *ab* as a centre. Now equal parallel forces applied by the cords (*ah* and *bi*) on each side of the pins (*ab* and *hi*) act in all cases like single forces applied at the centres of these pins (see Experiment 61, ¶ 159, 1). If the centre of gravity of the cylinder and disc is in the axis of *hi*, we have, as in the simple pendulum, a weight acting as if it were applied at a single point (in *hi*), and made by forces also applied at the same point (in *hi*) to oscillate about another point (in *ab*) as the centre. There is no rotation either of the cylinder or of the disc to complicate the result, as in the case of an ordinary compound pendulum. Evidently no such rotation can exist, unless the cords (*ah* and *bi*) slip on the pins (*ab* and *hi*). There is, moreover, no tendency to produce such rotation; because forces acting at the centre of gravity of a body (in *hi*) can cause only a linear motion of that centre of gravity. A line in the disc or cylinder which is vertical in one position of the pendulum, remains accordingly vertical in all positions. Here lies an essential distinction between this and other compound pendula.[1]

¶ 151. **Determination of Times of Oscillation by the Method of Coincidences.** — A pendulum between 100 and 101 *cm.* in length, adjusted and measured as in ¶ 150, is placed, let us say, in front of the pendulum of a regulator (Fig. 152, ¶ 149) and set in vibration in an arc not exceeding 10 *cm.* in length (that is, 5 *cm.* on each side of the vertical — see Table 3, *g*). Each swing will occupy a little over a second; hence the first pendulum will fall slowly behind the second. The two pendula will be moving now the same way, now opposite ways. The ticks of the regulator will occur when the first pendulum is now at its furthest right-hand or left-hand point, and now when it is at the middle point of its swing. Every such correspondence involves a "coincidence" of some sort. The object of this experiment is to find the average interval of time between two coincidences of a given kind. The student will be surprised to find in the reduction of different results (¶ 152) how large an error may be committed in the method of coincidences without introducing any considerable error into the result.

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[1] The student may notice that the time of oscillation of the stick used in Exp. 56 is considerably greater than that of a simple pendulum (see Table, ¶ 149) equal in length to the distance between the centre of gravity of the stick and its point of suspension. This is owing to the fact that gravity has not only to move the centre of the stick through a certain angle about its point of suspension, but also to turn the stick through the same angle. For a similar reason all ordinary compound pendula are somewhat retarded.

I. Ocular Method. — When the pendula are apparently swinging the same way, the time is to be read by the clock in hours, minutes, and seconds; and again the time is to be noted when the pendula seem to be moving in opposite ways. This should be continued for half an hour or more, according to the length of time that the pendulum may continue to swing perceptibly. The two pendula will probably seem to coincide for a long time in each case. Every effort must be made to determine the middle of such periods of coincidence.

II. Eye and Ear Method (§ 28). — The times may be noted when the ticks of the regulator are heard just as the pendulum under observation reaches its furthest point to the right or to the left; or better, when it reaches the middle point of its swing. In the latter method, the time of coincidence may be generally found within 10 seconds. It may be convenient in some cases to connect an electrical telegraph instrument with a break-circuit in the clock (Fig. 152, *a*) so that the ticks may be re-enforced or reproduced at a distance.

III. Optical Method. — Instantaneous views of the pendulum of the regulator may be obtained through the opening, *k*, in a fixed shield (Fig. 153), and an opening, *j*, in the disk of the pendulum. The regulator should be illuminated so that these views may produce a sufficient impression upon the eye. The times are to be noted when the pendulum of the regulator is seen at the middle point of its swing. Times of coincidence may thus be determined within a few seconds.

IV. Electrical Method. — An electrical current is sent first through the break-circuit of the clock (Fig. 152, ¶ 149), then through the break-circuit *lmno* (Fig. 156) attached to the pendulum (see Pickering, Physical Manipulation I. § 41). The ends of these wires should be amalgamated by dipping them first in nitric acid, then in mercury in order to make good electrical connections. The two hollows, *n* and *o* (Fig. 157), must be filled with mercury and raised by thin wedges so that the mercury may touch the wires (*lm*) in the middle point of the swing (*m*, Fig. 155).

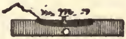

Fig. 155.

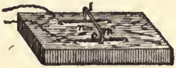

Fig. 156.

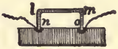

Fig. 157.

When the swings of the two pendula come into a certain mutual relation, an electrical connection will be made by both break-circuits at the same time, and the sounder will respond. After a certain time this relation will cease, and the sounder will become silent. The beginning and end of each period of response should be noted, and the middle of the period found by calculation. This method, though more complicated in detail, requires much less effort than the optical method, and is in general equally accurate.

The experiment is to be repeated with a *hollow* cylinder of sheet zinc, instead of the solid zinc cylinder represented in *gj*, Fig. 153; then again repeated with this hollow cylinder filled with sand or lead shot. The weights of the empty cylinder and its contents should be noted.

¶ 152. **Reduction of Results obtained by the Method of Coincidences.** — The reduction of results obtained by the method of coincidences will be best explained by an example. The times of coincidence should be arranged (see § 61) in three columns of about equal length. These columns should contain an odd number of observations, and should be averaged, thus: —

		min.	sec.		min.	sec.		min.	sec.
	1st	13	41	6th	24	0	11th	34	32
	2d	15	44	7th	26	3	12th	36	34
	3d	17	51	8th	28	9	13th	38	39
	4th	19	56	9th	30	15	14th	40	46
	5th	21	58	10th	32	23	15th	42	49
Average	3d	17	50	8th	28	10	13th	38	40

The first average corresponds in the example to the time of the 3d observation; the second average corresponds similarly to the 8th observation, and the last average corresponds to the 13th observation. For reasons stated in § 51, these averages are probably more accurate than the single observations to which they correspond. The difference between the first and second averages is 620 seconds; and since between the 3d and 8th observations, to which they correspond, there are 5 intervals, the average for each interval must be 124 seconds. It appears, therefore, that in 124 seconds the first pendulum loses just one swing with respect to the regulator; that is, it makes 123 swings while the regulator makes 124. Assuming that 124 swings of the regulator occupy as many seconds, one swing of the first pendulum must occupy $\frac{1}{123}$ of 124 seconds, or 1.0081 sec. In the same way, between the 8th and 13th observations, we find coincidences on the average 126 seconds apart; hence the average time of one swing is $\frac{1}{125}$ of 126 seconds, or 1.0080 sec. The student should note that the time occupied by one swing (1.0081 sec.) in the first part of the experiment differs very slightly from that (1.0080 sec.) in the last part of the experiment. The difference, due to a decrease in the arc of the pendulum, is in fact only about $\frac{1}{10000}$ of a second (see Table 3, *g*). He should also notice that this small difference in the result corresponds to a comparatively large difference (2 seconds) in the average interval between coincidences. Even with rough methods (¶ 151, I. and II.) such a difference could hardly fail to be observed when sufficiently multiplied by a *long series* of observations. If, conversely, the average interval between coincidences can be found within 2 seconds, the time of oscillation must be accurate within $\frac{1}{10000}$ of a second.

A comparison of results obtained with a solid and with a hollow cylinder of a given size and shape should show that the resistance of the air (which must exert a relatively greater influence in one case than in the other) is slight. A comparison of results obtained with a hollow pendulum filled with *different materials* should show that the time of oscillation of a pendulum of given length is independent of the nature of the substance of which it is composed.

¶ 153. **Relation between the Length and Time of Oscillation of a Pendulum and the Acceleration of Gravity.** — We have already seen (¶ 149) that a relation must exist between the length of a pendulum and the distance traversed by a falling body while the pendulum is making one swing. To find the distance which a body falls in 1.0081 sec. we have only to multiply the length of the pendulum, let us say 100.8 *cm.* by a certain number (4.935) already determined. From the distance which a body falls, and from the time occupied, we may calculate the velocity imparted to the body (see § 108); and from the velocity imparted in a given length of time, we can find that imparted in 1 second (§ 108). This is called the acceleration of gravity, and is denoted by g in the formulæ of § 108. To shorten this calculation, which depends solely on the length and time of oscillation of a pendulum, the following table has been computed for simple pendula between 99 and 101 *cm.* in length: —

TIME OF OSCILLATION.

Length of Pendulum.								
99.0	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970	0.9965
99.1	1.0006	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970
99.2	1.0011	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975
99.3	1.0016	1.0010	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980
99.4	1.0021	1.0016	1.0010	1.0005	1.0000	0.9995	0.9990	0.9985
99.5	1.0026	1.0021	1.0015	1.0010	1.0005	1.0000	0.9995	0.9990
99.6	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000	0.9995
99.7	1.0036	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000
99.8	1.0041	1.0036	1.0031	1.0025	1.0020	1.0015	1.0010	1.0005
99.9	1.0046	1.0041	1.0036	1.0030	1.0025	1.0020	1.0015	1.0010
100.0	1.0051	1.0046	1.0041	1.0035	1.0030	1.0025	1.0020	1.0015
100.1	1.0056	1.0051	1.0046	1.0040	1.0035	1.0030	1.0025	1.0020
100.2	1.0061	1.0056	1.0051	1.0045	1.0040	1.0035	1.0030	1.0025
100.3	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035	1.0030
100.4	1.0071	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035
100.5	1.0076	1.0071	1.0066	1.0061	1.0055	1.0050	1.0045	1.0040
100.6	1.0081	1.0076	1.0071	1.0066	1.0060	1.0055	1.0050	1.0045
100.7	1.0086	1.0081	1.0076	1.0071	1.0065	1.0060	1.0055	1.0050
100.8	1.0091	1.0086	1.0081	1.0076	1.0070	1.0065	1.0060	1.0055
100.9	1.0096	1.0091	1.0086	1.0081	1.0075	1.0070	1.0065	1.0060
101.0	1.0101	1.0096	1.0091	1.0086	1.0080	1.0075	1.0070	1.0065
$g =$	977	978	979	980	981	982	983	984

The length of the pendulum is to be found in the left-hand column; then in line with it the number nearest the time of oscillation is to be selected. Beneath this number, at the bottom of the column will be found the value of g.

EXAMPLE I. Given the length, 100.8 *cm*, and the time, 100.81 sec., required g. We find the time of oscillation, 1.0081, in the 4th column in line with 100.8 in the left-hand column and at the bottom of the 4th column we find the number 979, which represents the acceleration of gravity in question.

EXAMPLE II. Given the length, 100.84, and the time, 100.81, required g. We notice that the times increase by the amount .0005 when the length increases by 0.1 *cm.*; hence 0.04 *cm.* corresponds to .0002 sec.

If, therefore, the length had been 100.8 instead of 1.0084 the time would have been 1.0079 instead of 1.0081. Now 1.0079 comes between two numbers opposite 1.008, namely 1.0081 and 1.0076. Under the first we find 979, under the second we find 980. Since 1.0079 differs from 1.0081 by .0002 sec., and a difference of .0005 sec. makes a difference of 1 unit in g, we must add $.0002 \div .0005$ or $\frac{2}{5}$ of a unit to 979 to find the value of g. We have, therefore, $g =$ 979.4.

The object of this calculation is not so much to determine the value of g, which is already known with sufficient accuracy for all latitudes (see Table 47), and is believed to be the same for all materials, but rather to obtain a check upon the standards and methods hitherto employed for the measurement of length and time.

EXPERIMENT LIX.

INERTIA, I.

¶ 154. **Determinations of Mass by the Method of Oscillations.**—A small glass beaker (d, Fig. 158) is to be suspended from a support, a, by a coiled spring of steel wire, bc, as long and as flexible as may be convenient. A substance whose mass is to be determined is placed in the beaker. The beaker is then pulled downward to a position d', vertically beneath d, then released. It will spring up to a position d'', nearly as far above d as d' is below it. Then it will return nearly to d', and thus make a considerable number of oscillations before it comes to rest. The oscillations should not displace the load in the beaker; if they do, the load must be rearranged, or the oscillations must be diminished in amplitude. The time of oscillation is now to be found as in ¶ 149.

The load is next removed from the beaker, and in its stead weights from a set are placed there, sufficient in quantity to stretch the balance to the same point as before. The time of oscillation is again determined. If it is less than before, more weights are added, if greater, weights are removed; and thus by trial (§ 35) the weight is adjusted until the time of oscillation is the same with the weights as with the substance, the mass of which is to be determined.

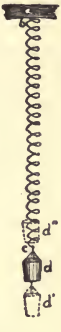

Fig. 158.

The student should notice that the time of oscillation is nearly independent of the amplitude of oscillation as in an ordinary gravity pendulum. It should be pointed out, however, that in the vertical oscillation shown in Fig. 158, gravity has nothing to do with the time of oscillation in question, except in so far as it may affect the elasticity of the spring by stretching it to a greater or less extent. When a spring is already loaded the force required to stretch it 1 *cm.* further may be taken as a measure of the stiffness of the spring under the load in question.

The time of oscillation of a load suspended by a spring depends (1st) on the stiffness of the spring and (2d) on the *mass* to be set in oscillation. When two loads give the same time of oscillation under the same circumstances, their masses are necessarily equal.

Having adopted as our standard of mass a certain piece of platinum in the French Archives (§ 6), we should theoretically use platinum weights in this experiment. It has been found, however, that two quantities which have equal masses, estimated as above by the dynamical method, have also equal weights (*in vacuo*); that is, gravity exerts the same acceleration upon them, without regard to the substances of which they are composed (see Exp. 58.) The use of brass weights will not, therefore, in *practice*, introduce any error.

The results of Exp. 59 are to be expressed in grams like results obtained by an ordinary balance. Strictly, however, the word *mass* should be written before or after these results instead of the word *weight* (§§ 152, 153).

¶ 155. **Relation between Weight and Mass.** — The student must not assume that weight and mass are *necessarily* the same. We do not know *why* a body is attracted by the earth, neither do we know *why*, being attracted, it does not move instantly, under that attraction, from one place to another. The former phenomenon we attribute to *gravity* (§ 150), the latter to *inertia* (§ 151).

By the weight in grams of a body we mean the number of grams of platinum to which the body is equal in respect to weight proper (§ 153), or the force exerted upon it by gravity. By the mass in grams of a body we mean the number of grams of platinum to which it is equal in respect to *inertia*, or the necessity of force to set it in motion (§ 152).[1] In the absence of any explanation of gravity and inertia, no reason can be assigned why any proportion should exist between them. There is no proportion between electrical or magnetic forces and the masses upon which they act. The existence of such a proportion between mass and weight is simply an inference from the results of experiment (see Exp. 58). It is possible, so far as we know, that a new substance may be discovered, the mass of which may be disproportional to its weight. It is also possible that if masses could be measured with the same accuracy as weights, slight variations might be discovered which have hitherto escaped observation. We have several instances of physical laws which are approximately but not exactly fulfilled; as for instance the law connecting the molecular weights and specific heats of elementary substances (§ 86, note). At the same time that such variations are *possible*, as far as we know, in the case of gravity and inertia, it is by no means *probable* that any such will ever be discovered. It is much more probable that gravity and inertia are both manifestations of a single principle, according to which, for reasons unknown to us, one must be proportional to the other.

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[1] See Hall's Elementary Ideas, published by C. W. Sever, Cambridge, Mass.

EXPERIMENT LX.

INERTIA, II.

¶ 156. **Determination of Force by Observations of Mass, Length, and Time.** — A metallic ring about 20 *cm.* in diameter, and weighing about 500 grams (C D F E, Fig. 159) is suspended horizontally by a spring brass wire *AB*, about 0.25 *mm.* in diameter (No. 31, B.W.G.), and at least one metre long. The wire is fastened at the top and held at the bottom by a small vice, *B*. This vice, *B*, is connected by fine iron wires (about No. 31) with four points C, D, E, and F of the ring. A paper millimetre scale is attached to the ring, and the distance through which it revolves is indicated by a fixed marker (*G*).

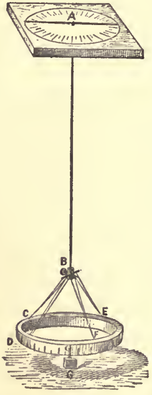

Fig. 159.

The reading of the marker is to be first observed when the ring is at rest. Then the ring is turned through nearly 360°, and released. All pendular vibration must be stopped by touching (if necessary) the wire *AB*. The ring will then have only a rotary movement, due to the "torsion" of the wire. As the ring approaches a turning-point, several readings of the marker are taken at intervals of two seconds. The intervals may be determined by the ticks of a regulator, or by an electrical sounder connected with the regulator.[1]

When the experiment has been repeated a sufficient number of times, the ring is taken down and its weight in grams determined. The vice, *B*, should not be weighed with the ring. It is better not to weigh the connecting wires with the ring; but their weight (which should not exceed 1 gram) will not in any case introduce a serious error into the result. The material, length, and diameter of the wire *AB* should be noted. The observations are then to be reduced as in ¶ 157.

¶ 157. **Calculation of Force from Observations of Mass, Length, and Time.** — The rotation of a ring about its axis presents one of the simplest cases in dynamics. The whole mass of the ring is at (nearly) the same distance from the axis in question, and hence acquires (nearly) the same velocity. To find the force exerted upon the ring in the direction of this velocity, we have to find (1) the velocity acquired, (2) the time required to attain this velocity, and (3) the mass acted upon. The force may then be calculated by the general formula (§ 106): —

$$f = \frac{mv}{t}$$

---

[1] If greater precision is required than can be obtained by the eye, a small bristle attached to the armature of the sounder can be made to mark the seconds on the edge of the ring, which must be previously smoked for this purpose. By employing two such markers on opposite sides of the ring, slight errors due to swinging of the ring can be eliminated.

In practice we make this calculation as in the example below. The observations are numbered and arranged as follows: —

	mm.	Difference in 2 sec.	Mean Velocity.	Difference in 2 sec.	Acceleration
1	552			—	—
		+33	+16.5		
2	585			8.0	4.0
		+15	+ 7.5		
3	600			10.0	5.0
		− 5	− 2.5		
4	595			7.5	3.8
		−20	−10.0		
5	575			10.0	5.0
		−40	−20.0		
6	535			—	—

The differences in the 3d column show the distance passed over in 2 seconds; hence these are divided by 2 to find the distance passed over in 1 second, or the mean velocity for a period of 2 seconds. The velocity is called positive if the ring is turning away from its position of equilibrium, otherwise negative. The 5th column shows the algebraic differences in these velocities; that is, the change of velocity in 2 seconds. To find the acceleration, or change of velocity in one second, the numbers in the 5th column must be divided by 2. This gives the numbers in the 6th column, the average of which is 4.5, nearly. Since we have used *mm.* throughout, the change of velocity in one second amounts to 4.5 *mm. per sec.*, or 0.45 *cm. per sec.*

This is the acceleration strictly of the outer surface of the ring. Let us suppose that the outside diameter is 20.5 *cm.* and the inside 19.5 *cm.*, so that the mean diameter is 20.0 *cm.*; then the average acceleration will be less than 0.45 in the ratio of 20.0 to 20.5. The average acceleration will be, therefore, about 0.44 *cm. per sec.* If now a mass of 500 *g.* receives this acceleration, the force exerted upon it must be 500 × .44, or 220 dynes (§ 12). The angle through which the steel wire is twisted is given in circular measure by the ratio of the arc to the radius. Since the latter is 10 *cm.* (nearly), the minimum deflection (53.5 *cm.*) corresponds to 5.35 units of angle. The maximum deflection (60.0 *cm.*) corresponds similarly to 6.00 units of angle. The mean deflection is accordingly not far from 5.7 units of angle. Since one unit of angle in circular measure is equal to 57°.3, nearly, the mean deflection of the ring is about 57°.3 × 5.7, or 327°.

We note, therefore, that a piece of steel wire of given length and diameter, when twisted 327°, exerts at a distance of 10 *cm.* from its axis a force of about 220 dynes.

The use which is to be made of this result will be explained in ¶ 165 in connection with a method by which a force similar to the one in question may be directly balanced by gravitation. A more accurate method of reducing results obtained by the "torsion pendulum" will be given in the Appendix (Part IV).

EXPERIMENT LXI.

COMPOSITION OF FORCES.

¶ 158. **Correction of Spring Balances.** — A spring balance consists of a spiral spring, *cd* (Fig. 160), contained in a hollow metallic case, *bh*, to which it is fastened at *c*. The spring is connected by a rod, *di*, with a hook, *ij*, from which weights are hung. A slit, *eg*, is made in the case so that a pointer, *f*, attached to the rod, *di*, may indicate the elongation of the spring on a scale outside of the case. In measuring vertical forces with a spring balance, the instrument is generally suspended by the ring, *a*. When forces in other directions are to be determined, the case (*bh*) should also be supported, so as not to bear against the index, *f*. If this precaution is not observed, large errors from friction may be introduced into the results. Spring balances are usually graduated so as to indicate the weight of a body either in kilograms or in pounds. It must be remembered that such indications are affected by the force of gravity. Thus a spring balance, graduated correctly in England, would give, in Brazil, readings too low by about $\frac{1}{3}$ of 1 %. Obviously spring balances, however sensitive, cannot serve everywhere as standards of *mass* (§ 6). The readings depend, not directly upon the masses suspended, but upon the forces which they exert on the instrument. A spring balance once graduated correctly in *megadynes*[1] should, however, give *forces* correctly (in megadynes) irrespective of locality. A

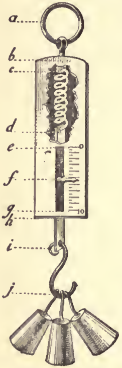

Fig. 160.

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[1] The student may be interested to cut a scale of megadynes by the side of the ordinary scale. In latitude 40°–45°, 1 megadyne = 1.02 kilos. = $2\frac{1}{4}$ lbs. nearly.

spring balance is essentially an instrument for measuring force, and it is only *in a given latitude* that it may be employed for estimating weights either in kilograms or in pounds. A pair of 10-kilo. (or 24-lb.) spring balances will be suitable for the experiments which follow.

The reading of a spring balance may be corrected by hanging known weights upon it, as in Fig. 160. Weights provided with a ring, a hook, or an eye will be found convenient for this purpose. The reading of the balance should be tested with weights of 1, 2, 3, etc., up to 10 kilos. (or 2, 4, 6, up to 24 lbs.). The zero-reading of the spring balance should also be found, both in a vertical and in a horizontal position. The weights used may be compared by an ordinary balance with standards if it is thought necessary. From these results we are to calculate the corrections to be added to the reading of the spring balance under different loads, in order to find the true load. Thus if the indication with a 4 lb. weight is 3 lbs. 14 oz., the correction is +2 oz. The results should be arranged in tabular form, either in kilos. or in pounds, as follows: —

FIRST TABLE OF CORRECTIONS.

(1) Load in kilos.	Correction in kilos.	(2) Load in lbs.	Correction in oz.
0	−0.10	0	−3
1	−0.05	2	−1
2	+0.08	4	+2
3	+0.25	6	+6
. . .	. . .	. . .	. . .
10	+0.05	24	+1

One of the weights is now to be attached to the spring balance by a light but strong cord (*ac*, Fig.

161) passing over a pulley (*b*) made to run as freely as possible. The readings of the balance are to be carefully compared in different positions (a', a'', etc.). To eliminate the effects of the friction of the pulley, the readings are to be made in each case (1) when the weight is being slowly raised, and (2) when it is being slowly lowered. If the two readings differ perceptibly, the mean is to be taken.

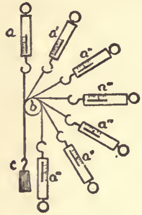

Fig. 161.

The object of testing a spring balance in different positions is to eliminate the effects due to the weight of the hook and spring.[1] From the results we are to calculate the corrections to be added to the readings under different inclinations in order to find the reading in the vertical position. Thus if a 2 lb. weight weighs apparently 2 lbs. 1 oz. in the vertical position, and 1 lb. 11 oz. in the horizontal position, the correction for an inclination of 90° is +6 oz. These corrections should be the same for all weights, and should be entered in a second table, as follows: —

SECOND TABLE OF CORRECTIONS.

(1) Inclination of Balance.	Correction in kilos.	(2) Inclination of Balance.	Correction in oz.
30°	+0.02	30°	1
60°	0.08	60°	3
90°	0.16	90°	6
120°	0 24	120°	9
150°	0.30	150°	11
180°	0.32	180°	12

[1] This method was suggested to the author by a similar one employed by Mr. Forbes of the Roxbury Latin School. See also Elementary Physical Experiments, published by Harvard University, page 11, footnote.

¶ 159. **Determinations of Weight by the Composition of Forces.** — It is frequently inconvenient to measure the weight of a body directly, either by ordinary scales, or by a single spring balance, as when the weight of the body exceeds the capacity of such instruments, or when the body forms an inseparable part of a combination. In such cases, we may sometimes make use of principles involved in the composition and resolution of forces.

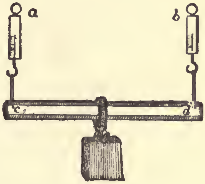

Fig. 162.

(1) To find the force of gravity on a "28-lb." weight with two spring balances, each of 10 kilograms' capacity, we hang the weight (*e*, Fig. 162) at the middle of a stick (*cd*) so that it may bear about equally upon the spring balances (*a* and *b*) while hanging in a vertical position. The reading of each balance is to be noted; then the weight is to be removed, and the readings again taken with the stick alone. The difference between the two readings of a given balance, with and without the weight, corrected if necessary by Table I., ¶ 158, gives the part of the load borne by that balance. The sum of the two parts is of course equal to the whole load.

(2) To find the force of gravity on a "56-lb." weight with a single spring balance of 10 kilograms' capacity, we suspend a lever (*cd*, Fig. 163) as before, except that a cord, *bd*, takes the place of the spring balance (*b*, Fig. 162). The weight is then hung at a

point, *e*, let us say one-fourth the distance from *d* to *c*, and the reading of the spring balance is observed. Care must be taken that the cords *fg* and *hi*, by which the weight is suspended, swing free of the side of the lever as in the cross-section (Fig. 163). A similar precaution should be observed in respect to the cords by which the spring balance, *a*, is attached to the lever at *c*. The cords should both be vertical. The horizontal distances *cd* and *ed* are to be accurately measured. The weight is now to be removed, and the reading of the spring balance again noted. If *F* and *f* are the forces indicated by the spring balance with and without the weight, both being corrected by the first table of ¶ 158, the force (*w*) exerted by the weight at *e* is evidently equal to $F - f$. If we call the whole weight *W*, then since the *couple* (§ 113) produced by *W* (equal to $W \times de$) is balanced by the couple produced by the spring balance (equal to $w \times cd$), allowing for the weight of the lever, it follows that —

$$W = (F - f) \times cd \div ed.$$

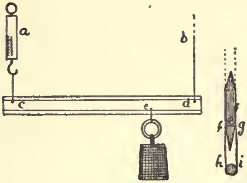

Fig. 163.

(3) Another method of suspension is represented in Fig. 164. It is assumed that the weight will be able to lift the lever, so that the balance must be applied from under the lever. The reading of the

balance in this position must be corrected both by the first and by the second table of ¶ 158. Thus since the inclination of the balance is 180° (compare Figs. 164 and 161), we must add 0.32 kilos according to the second table (¶ 158), *besides* the ordinary correction for the observed reading from the first table (¶ 158). In addition to the force exerted by the spring balance, we have that part of the weight of the lever which is felt at *a*, helping to balance the 56-lb. weight. To allow for the weight of the lever, we remove the 56-lb. weight, and apply the spring balance as in Fig. 163, so as to sustain the lever at *a*. The reading of the balance in this position needs to be corrected simply by the first table (¶ 185), and gives the force (f) exerted by the lever at *a*. This is to be *added* accordingly to the force (F) exerted by the spring balance with the weight (e) to find the total force which balances this weight. Calling this force w, and the load W, we have $w \times ab = W \times bc$, or —

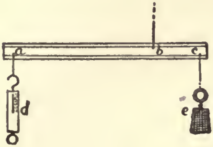

FIG. 164.

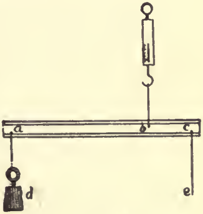

FIG. 165.

$$W = (F + f) \times ab \div bc.$$

(4) To test a 4-lb. weight with a 10-kilogram spring balance, we fasten one end of a lever (c, Fig. 165) to

the ground by means of a vertical cord, *ce*, and suspend the lever from a spring balance by a cord *b*, not far from *c*. The force, *f*, indicated by the balance is to be observed. The weight, *d*, is then hung from the free end of the lever, and the force (F) indicated is again observed. Allowing as before for the weight of the lever we find the force ($F - f = w$) exerted by the spring which balances the load W at *d*. Then since $W \times ac = w \times bc$, we have $W = (F - f) \times bc \div ac$.

If the distance *bc* is one fourth of *ac*, every ounce at *a* will produce an effect at *b* equal to 4 oz. We might therefore weigh a small object to ounces with a balance graduated only to 4 oz. (or $\frac{1}{4}$ lb.).

(5) Another method of weighing small objects is to hang two spring balances, *A* and *B* (Fig. 166), from

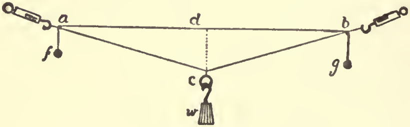

Fig. 166.

nails in the wall, 2 or 3 metres apart, then to connect them by a cord *acb*. At the middle of the cord (*c*) a ring (*C*) is hung so that the weight, *W*, may be readily attached. Two pins are driven into the wall opposite points *a* and *b*, on the cords at equal distances (let us say just 1 metre) from *c*. A cord, *ab*, is stretched between them by means of two small weights, *f* and *g*. The perpendicular distance, *cd*, between *c* and *ab* is then measured.

The vertical component of the force A registered by the spring balance near a, is by the triangle of forces (§ 105) equal to $A \times cd \div ac$. The vertical component of the force, B, due to the spring balance near b, is similarly $B \times cd \div bd$. The total sum of these components must balance the combined weight of the ring (C) and of the load (W). That is,

$$W + C = A \times cd \div ac + B \times cd \div bc.$$

To eliminate the weight of the ring, the load (W) is removed, and the experiment is repeated with the ring alone, as in Fig. 167. We have, similarly,

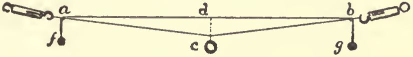

FIG. 167.

$$C = A \times cd \div ac + B \times cd \div bc.$$

Hence subtracting the last value (C) from the first ($W + C$) we find the weight of the load (W) in question.

We will assume, for simplicity, that a and b are on the same level. A slight difference in level will, however, have no appreciable effect upon the result. The sagging of the cord ab will probably be very small, and will be eliminated in the method of difference by which the result is calculated.

The same method may be employed for the measurement of large weights. If the angle acb is small (see Fig. 168), it will be more accurate to calculate cd from a measurement of ab, than to measure cd directly. Let us suppose that the cords bB and aA have been lengthened or shortened so that the line ab is horizontal. The vertical line cd will then be at right angles with ab; and since $ac = bc$, $ad = bd = \frac{1}{2}ab$. Knowing ad, we may calculate cd by the Pythogorean proposition —

$$cd = \sqrt{(ac)^2 - (ad)^2},$$

and hence find the load C or W as before.

This method would be adopted in practice if for any reason it were inconvenient to obtain a point of suspension directly above the weight. We should prefer, however, to employ a lever long enough to reach, as in (1) or (2), between two available points of suspension, A and B, if it were possible to obtain one of suitable weight and strength.

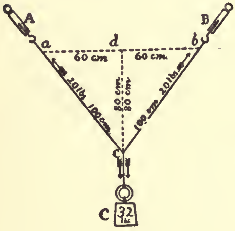

Fig. 168.

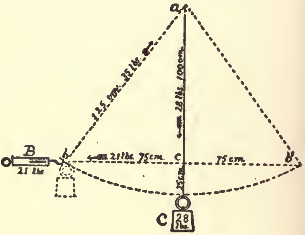

Fig. 169.

(6) To measure a weight (C, Fig. 169) when suspended by a cord (ac) we may pull it one side by a spring balance applied horizontally in the direction cb. The reading of the balance (corrected by *both* tables of ¶ 158) gives the force B acting in the di-

rection *cb*. This with the force of the cord acting in the direction *ba* produces a resultant which balances the weight of the body C. The direction in which the weight C acts must be parallel to that of the cord *ac* before the weight was disturbed. Since three forces in equilibrium are proportional (§ 105) to the sides of a triangle to which they are respectively parallel, we have $B : C = bc : ac$, or

$$C = B \times bc \div ac.$$

Instead of measuring *bc* directly, we may pull the cord *ac* first one side to a point *b*, then in the opposite direction to a point b' at a (nearly) equal distance from *c*. These points may be marked by pins, *b* and b' driven into the wall or into some other support behind the cord. The distance between *b* and b' is then measured and divided by 2 to find the distance *bc*. The point *c* may be found by a thread stretched between the pins *b* and b'. In this case the distance *ac* may be directly measured. Or the distance *ab* may be found and *ac* calculated (since *ab* is known) by the Pythagorean proposition,

$$ac = \sqrt{(ab)^2 - (bc)^2}.$$

By the use of very small deflections, we may measure weights many times exceeding the capacity of the spring balances which we employ.

EXPERIMENT LXII.

CENTRE OF GRAVITY.

¶ 160. **Location of the Centre of Gravity.** — A flat board,[1] *bcde* (Fig. 170), is suspended by a thread *abb′a′* (Fig. 170, 1) passing through a fine hole *bb′* in the board, and over a peg *aa′*. A plumb line, *af*, is also suspended from the same side of this peg, so as to hang as close to the board as possible. A projection of this line upon the board is to be traced in pencil (Fig. 170, 2). The eye must be held in this process so as to look perpendicularly upon the board (§ 25). The board is then to be hung by another point, *d* (Fig. 170, 3), and another line drawn upon it. Then the board is to be suspended from a third point, *c* (Fig. 170, 4), and a third line traced. All three lines

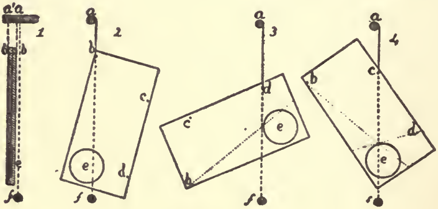

Fig. 170.

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[1] To lend interest to this experiment the board may be made of two thicknesses glued together, with a space (*e*, Fig. 170) between them which has been hollowed out and filled with lead. An irregularly shaped board may also be employed.

should intersect at a point in the surface of the board directly in front of the centre of gravity. If they do not, the experiment must be repeated.

¶ 161. **Determination of Weight by Displacement of the Centre of Gravity.** — A weight (w, Fig. 171) is attached at a to one end of a board whose centre of gravity (c) has been located (¶ 160); and the board is balanced upon a triangular piece of wood (d) or upon a pencil. The line of the support (bb' Fig. 172) is then marked upon the board, and two lines, ab and cb' are drawn from a and c perpendicular to bb'. These lines are then carefully measured. If W is the weight of the board, which we may consider as if concentrated at c (§ 112), we have $W \times b'c = w \times ab$; whence $W = w \times (ab) \div (b'c)$.

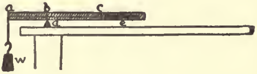

Fig. 171.

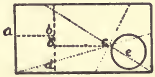

Fig. 172,

The experiment should be repeated with different weights applied at different parts of the board, and with the line bb' not always at the same place or in the same direction. The different values calculated for the weight of the board should be averaged. From their agreement we may infer the truth of the assumption that the weight of a body acts in all cases as if applied at its centre of gravity.

It is obvious that if W and w are both known, we may calculate the distance ($b'c$) by the formula

$$(b'c) = w \times (ab) \div W.$$

To find the distance of the centre of gravity from an axis (bb') on which a body balances, it is only necessary to know the weight of the body (W), the load (w), and its distance (ab) from this axis. For an experiment (due to Prof. Hall) in which this principle is applied, see Ex. 17 of the Elementary Physical Experiments, published by Harvard University.

EXPERIMENT LXIII.

BENDING BEAMS.

¶ 162. **Determination of the Stiffness of a Beam.** — A square steel rod, *ag* (Fig. 173), is mounted on two triangular supports with steel edges, *i* and *j*, 1 metre apart. A screw with a micrometer head (d) is adjusted so that its point just touches the middle of the beam when a pan, m, is suspended from it by the wires hk. The micrometer is then read. A load, l, is next placed in the pan, and the micrometer is once more adjusted until it touches the beam. The micrometer is again read. Its point is then withdrawn,

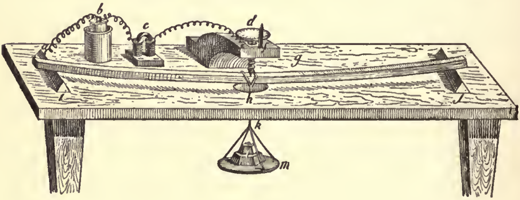

Fig. 173.

so as not to be injured by the recoil of the beam when the weight is removed. A new reading is then taken with the pan (m) empty. If this differs greatly from the first, the beam has probably been permanently bent, and the experiment must be repeated with a smaller load. If the reading is the same as before, a larger load may be tried. With a steel beam 100 *cm.* long and not over 1 *cm.* thick, a deflection of several centimetres should be possible without injury to its power of recovery. To discover exactly when the point of the micrometer touches the beam, we may make use of an electrical contact. One pole of a voltaic cell, b, is to be connected with one end of the beam by a wire soldered to it at a. The other pole is connected with one binding post of an electrical sounder c. The other binding post of this sounder is connected by a wire with the metallic nut e, in which the micrometer turns. The point of the micrometer and the surface of the beam beneath it are scraped bright with a file (or better, coated with platinum). When the point of the micrometer touches the beam, the electrical circuit $bceab$ is thus completed, and the armature of the sounder is attracted. A motion of one thousandth of a millimetre is sufficient, under favorable circumstances, to make or break the contact.

Care must be taken to prevent the beam from twisting or rocking under the influence of a load. The load should not bear more heavily on one side of the beam than on the other. Both sides should be supported alike at each end of the beam by the sharp edges *i* and *j*. Various deflections under different loads are now to be determined. Each deflection requires two readings of the micrometer, one with, the other without the load. The distance between the supports *i* and *j* should be measured with a metre rod, and the breadth and thickness of the beams employed should be determined at different points with a micrometer gauge (¶ 50, II.).

(1) The deflection of a beam, let us say 1 *cm.* square, is first to be determined with the supports (*i* and *j*, Fig. 173) exactly 100 *cm.* apart, and with a load causing the greatest deflection which can be employed without permanently bending the beam, or exceeding the reach of the micrometer.

(2) The deflection due to one half this load is next to be found. The student should notice that this deflection is almost exactly half as great as before (see § 115). If it is not, the measurements in (1) and (2) should be repeated. The same should be done if the zero-reading of the micrometer is changed.

(3) To test the stiffness of the middle portion of the beam, the supports *i* and *j* are to be placed 50 *cm.* apart, — that is, with half the original distance between them. The rod is to be mounted upon them as before, but with 25 *cm.* or more at either end projecting beyond the supports. The beam is to be loaded with 4 times the weight used in (1) or 8 times that used in (2). If the beam is equally stiff in all parts, the deflection should now be the same as in (2). (See § 115.)

(4) The experiment is next to be repeated with the supports 100 *cm.* apart, with a beam twice as broad as the one first employed, but having the same thickness and bearing the same load as in (1). If the material of the beam is the same as in (1), the deflection due to a given weight should be the same as in (2), since the breadth and weight have the same relative proportion as in (2).

(5) The beam is now to be turned edgewise, and loaded as in (3). The deflection is to be determined as before. If the depth of the beam is just twice as great as in (2), and the width the same, since the force employed is eight times as great as in (2), the deflection should be the same as in (2).

¶ 163. **Calculations relating to Flexure.** — By five measurements arranged as above, we are able to test (in a single instance in each case) the application of the laws of flexure stated in § 115. These laws may be combined in a single formula. If l is the length of a beam, b its breadth, t its thickness, and d the deflection produced (all in *cm.*) by the force f (in dynes) exerted by the load; and if F is the force necessary to produce a unit deflection in a beam of unit length, breadth, and depth (supposing such a deflection to be possible), we have —

$$F = \frac{fl^3}{bdt^3}.$$

The quantity F is sometimes called the modulus of transverse elasticity. Knowing this modulus, we may evidently compute any one of the five quantities, f, l, b, d, or t, if the other four are known. The student should calculate the value of F from at least one set of measurements. He should also find, by the rule of simple proportion, what force would be required to produce a deflection of 1 *cm.* in the case of each beam which he has employed. Thus if, with a given beam, 1 kilogram produces a deflection of 2 *cm.*, 500 grams would be the force required to produce a deflection of 1 *cm.*

The force (500 grams in this case) producing a unit deflection may be taken as a measure of the *stiffness*[1] of the beam in question. The stiffness of a beam is due to the fact that in order to bend it, the under part must be stretched and the upper part squeezed or compressed. The forces brought into play by stretching will be measured directly in Experiment 65.

EXPERIMENT LXIV.

TWISTING RODS.

¶ 164. **Effect of Couples.** — An instrument serving both to measure and to illustrate the effect of different "couples" (§ 113) is shown in Fig. 174. It consists of a rod of ash (*ej*) 1 *cm.* square, driven into a square hole in a block (*j*) which is fastened to the floor. The rod passes through a large hole in a table to a circular disc of wood (*cg*) 20 *cm.* in diameter, at the centre of which is a square hole (*e*), into which the upper end of the rod is tightly fitted. Two markers, *b* and *g*, measure the rotation of the disc by means of a scale of degrees graduated on the edge of the disc. At certain points of the disc (*abc defgh*, Fig. 175), small screw-eyes are placed so that forces may be applied by cords attached to spring balances (*a* and *h*, Fig. 174). It is convenient that four or more of the points (*cdef*, Fig. 175) should be in the same straight line and at equal distances, let us say 6 *cm.* The points *a*, *b*, *g*, and *h* (Fig. 175) may be placed so that *ad*, *dg*, *be*, and *eh* are at right angles to *cf*, and each 8 *cm.* long. This will make the diagonal distances *ac*, *bd*, etc., each 10 *cm.*

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[1] Stiffness must not be confounded with breaking strength. A thin beam, though more easily broken than a thick one, is not so in proportion to its flexibility; for by reason of its thinness it can bend much *farther* than a thick beam without breaking. Both the strength and stiffness of a beam are proportional to its breadth; but the former depends upon the square of the ratio which the thickness bears to the length, while the stiffness depends upon the cube of this ratio. (See formula above.)

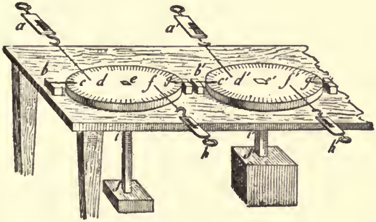

FIG. 174.

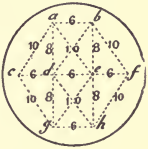

FIG. 175.

A very slight force applied at any point of the disc will cause the rod *cj* (Fig. 174) to bend so as to touch one side of the hole in the table. To keep the rod in the middle of this hole throughout this experiment,[1] equal and opposite forces must be applied to the disc. If these forces are applied at the same point, no effect will be observed. For instance, two equal forces applied at *d* (Fig. 175) in the directions *dc* and *de* (or in the directions *da* and *dg*) will neutralize each other. Again, if the forces and their points of application are all in the same straight line, the effect will be zero. Thus a force applied at *d* in the direction *dc* will offset an equal force applied at *e* in the direction *ef*. When, however, the lines in which the two forces act are parallel but not coincident, the couple which results (§ 113) will twist the rod. The angle through which the rod is twisted should be proportional to the magnitude of the couple acting upon the disc. The magnitude of the couple is equal (see § 113) to the product of either of the two forces which constitute it, and the "arm" or perpendicular distance between the lines in which the forces act.

The student should satisfy himself that it makes no difference where the "arm" is situated. Thus two opposite forces of 1 kilogram each applied at *a* and *b* or at *c* and *d*, at right angles to *cf*, will have the same effect as if applied in the same manner at *d* and *e*, respectively. The student will notice, moreover, that the rod is twisted but *never bent* by a pair of equal and opposite forces, whether these be applied at equal or unequal distances from the *centre* of the disc. He should also satisfy himself that with a given arm (as for instance *de*), the rod is twisted through an angle which is proportional to the forces employed (let us say 1, 2, or 3 kilograms); and that the twists produced by given forces (*e. g.*, 1 kilogram each) are proportional to the arms to which they are applied. Arms of the following lengths may be most conveniently employed: 6 *cm.* (*ab*, *cd*, *de*, *ef*, or *gh*); 8 *cm.* (*ad*, *be*, *dg*, or *eh*); 10 *cm.* (*ac*, *ae*, *bd*, *bf*, *gc*, *ge*, *hd*, or *hf*); 12 *cm.* (*ce* or *df*); 16 *cm.* (*ag* or *bh*); and 18 *cm.* (*cf*). Two *equal* forces must be applied in all cases in directions at right-angles to the arms, parallel to the disc, and opposite to each other. They should be made to twist the rod sometimes to the right and sometimes to the left.

---

[1] In trying this experiment, several students should work together. One may hold and read one of the spring balances, another the other spring balance, while a third observes the deflection of the disc.

To measure accurately the angles through which the disc rotates, both markers (*b* and *g*, Fig. 174) must be observed. It is easy to calculate from a given case by simple proportion what couple would be required to twist the rod through 1°. This gives us a measure of the stiffness of the rod under torsion which may be called its coefficient of torsion.[1]

We next employ a rod, *e′j′*, of half the length of *ej* (Fig 174). This rod must be mounted on a block (*j′*) much higher than *j*. We shall find, if the material and the cross-section are the same, twice the coefficient of torsion. If we use a rod of same length, having, however, twice the diameter, we shall find a coefficient of torsion 16 times as great as before (see Laws of Torsion, § 116). It is therefore important to measure and note the length and diameter of the rods employed.

---

[1] The coefficient of torsion must not be confounded with the strength of a rod to resist *fracture* by torsion. See note ¶ 163.

We shall apply the principles illustrated in this section to the determination of the coefficient of torsion of a wire.

¶ 165. **Determination of the Coefficient of Torsion of a Wire by means of a Torsion Balance.** — A hard drawn brass wire about 2 metres long and 0.25 *mm.* diameter (about No. 31, B.W.G.) is stretched horizontally between a knitting-needle (*bd*, Fig. 176) and a fixed support (*k*). The joints should be soldered both at *c* and at *k*, or made equally firm in any other manner.

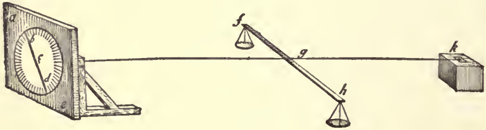

Fig. 176.

The knitting-needle is held in place by a paper protractor fixed on the surface of a board (*ae*). The board and protractor are pierced at the centre (*c*) so that the wire may pass through. A thin strip (*fh*) of some light wood, 20 *cm.* long, is attached at its central point, *g*, to the middle of the wire by sealing-wax. From the ends of this strip two paper scale-pans are suspended by threads. The "torsion" balance thus constructed should not weigh more than one or two grams.

The knitting-needle is first set so that the beam (fh) is horizontal. To do this, the beam must be sighted with reference to the bars of a window, or other horizontal line in the room. The reading of the needle is then found by observing both ends. This is the zero-reading of the instrument. Then a decigram is placed in one of the scale-pans, and the needle is turned until the beam is again horizontal. The decigram is then removed from the scale-pan, and the zero-reading re-determined. If any marked change has occurred, the experiment must be repeated. If the zero-reading is again disturbed, a weight smaller than 1 decigram should be employed.

The weight is to be placed first in one scale-pan, then in the other. In each case we note the angle through which the needle must be turned to the right or to the left from its zero position in order that the beam may be made horizontal. It is well to observe the zero-reading after the experiment, since the constancy of this reading is the only safeguard against slipping of the joints or permanent straining of the wires.

Since the balance beam is 20 *cm.* long, the average length of each arm must be 10 *cm.* Since the weight of 1 gram is about 980 dynes, that of 1 decigram will be about 98 dynes; hence the couple exerted by gravity is 98×10 or 980 units. This is balanced by twisting a certain portion of the wire (eg) through an observed number of degrees; hence the couple due to 1° is easily calculated. This couple measures a coefficient of torsion of the wire (see ¶ 164), which will be needed in experiments later on.

We notice that the portion of the wire between *g* and *k* is not twisted at the times of making our readings, because the beam *fh* remains horizontal. The torsion of this part of the wire does not, therefore, affect the result. The only use of the wire between *g* and *k* is to keep the balance in place. The length of the wire between *c* and *g* should be measured, and its diameter should be found in several places by means of a micrometer gauge (¶ 50, II.). The material should also be noted, in order that we may utilize our results in certain other experiments later on.

EXPERIMENT LXV.

STRETCHING WIRES.

¶ 166. **Young's Modulus of Elasticity.** — A fine steel wire, about 0.25 *mm.* in diameter (No. 31, B. W. G.) and 1 metre long, may, if made of the best steel, be stretched 1 *cm.* without breaking, or losing its power of recovery. We will suppose such a wire to be held at one end by a small vice (*a*, Fig. 177) and attached at the other end (*b*) to a spring balance (*c*) held in place by a nail (*d*). Let the read-

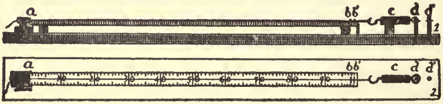

Fig. 177.

ing of this balance be 0. Now let the wire *ab* be stretched to a point b', by placing the balance over a nail (d), and let the new reading of the balance[1] be F. Then if the length of the wire thus stretched is ab centimetres and the elongation is bb' *cm.*, the stretching of 1 *cm.* will be $bb' \div ab$. This is called the *strain* of the wire. When 100 *cm.* are stretched, for instance, 1 *cm.*, we have a strain of 1 per cent or $+ .01$.

Now if the diameter of the wire is measured by a micrometer gauge, and divided by 2, we have its radius, r. From this we can find the cross-section q by the ordinary formula ($q = \pi\, r^2$), or

$$q = 3.1416 \times r^2, \text{ nearly.} \qquad \text{I.}$$

The cross-section can also be determined by finding the weight, w, of a given length (l) of the wire, if its density (d) is known; for since the volume of a wire is equal to $q \times l$, we have by definition (§ 154) $d = w \div ql$, whence —

$$q = \frac{w}{ld} \qquad \text{II.}$$

We will suppose that by either of these formulæ the average cross-section of the wire *ab* has been found. Now let the force indicated by the spring balance be *reduced to dynes* by multiplying by the appropriate factor.[2] Let us call this force in dynes f.

---

[1] In practice a small force will be required to straighten the wire. In this case the force F, below, must be taken as the difference between the forces exerted by the balance at d and d'.

[2] Thus in latitude 50° 1 kilogram is equal to about 981,000 dynes, 1 lb. avoirdupois to 445,000 dynes, and 1 oz. to 27,800 dynes, nearly.

To find the intensity of the force per square centimetre of cross-section of the wire, we divide it by the cross-section in question. Thus if the wire had a cross-section of one 2,000th of a square centimetre (.0005 cm^2), a force of 5,000,000 dynes would represent an intensity of 10,000,000,000 dynes per square centimetre (since $5,000,000 \div 0005 = 10,000,000,000$). The result is called the "*stress*" exerted upon the wire (§ 22).

It has been stated (§ 114) that for a given material there is always a certain proportion between the *stress* exerted upon it and the *strain* produced. The ratio of the stress to the strain in the stretching of a rod or wire is called "*Young's Modulus of Elasticity.*" If, for example, a stress of 10,000,000,000 dynes per square centimetre produces in a steel wire an elongation of one half of one per cent, that is, a strain of $+.005$, the Modulus of Elasticity of the steel is $10,000,000,000 \div .005$, or 2,000,000,000,000 (two millions of millions) *dynes per square centimetre.* The Modulus of Elasticity has also been defined as the *force* necessary (under Hooke's law, § 114) to produce a unit strain in a rod of unit cross-section; that is, *to double the length of the rod.* Evidently, if 10,000,000,000 dynes are required as above to increase the length of a steel rod, 1 *cm.* square, by one part in 200, it would take 200 times as much force to double its length, provided that it kept on stretching at the same rate; hence we find 2×10^{12} for the modulus of elasticity, as before.

Few substances can be stretched one hundredth part of their length without breaking. It is only in the case of exceedingly elastic substances, like India rubber, that the conditions suggested by the last definition can be actually attained. In the case of most substances, we can only calculate by the rules of simple proportion what stress *would* double their length, provided that fracture or other changes did not occur.

The student may notice that steel (see Table 9) has the greatest modulus of elasticity of any known substance, because it requires the greatest force to produce a given amount of stretching; or because, in other words, it yields the least. A substance like India rubber, which is in the ordinary sense particularly elastic, has for this very reason a small *modulus* of elasticity.

¶ 167. **Determination of Young's Modulus of Elasticity.** — The data necessary for a determination of Young's Modulus are, as will be seen from ¶ 166, (1) the length, (2) the cross-section of the wire to be tested, (3) the elongation produced in it by a given force, and (4) the magnitude of this force. The length of a wire may be measured, without any special difficulty, by a tape graduated in millimetres. The cross-section requires much greater care, whether it be determined (as suggested in ¶ 166) by measurements taken with a micrometer gauge at different points, or by its length, weight, and density. The principal difficulty consists, however, in measuring accurately the elongation of the wire, which is usually a very small quantity. To make the elongation as large as possible, long wires are usually employed.

One of the chief sources of error in measuring the elongation of a wire under a given load is due to the yielding of the support to which the wire is attached. Various devices have been suggested by which this effect may be eliminated. The simplest is to measure the distance between two points on the wire. This may be easily done, when a double wire is employed, by means of two micrometers, *a* and *b* (Fig. 178, 1), attached to the wall, and adjusted so as to touch two cross-bars borne by the wires in question.[1]

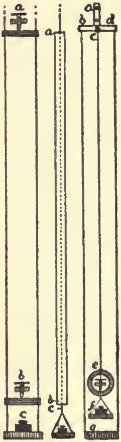

Fig. 178.

To avoid the inconvenience of making observations at a considerable height above the floor, a wire is sometimes surrounded by a tube (*ab*, Fig. 178, 2) attached to it at a point *a*. If the point *a* yields, a point *b* at the base of the tube will yield by an equal amount. The height of this point (*b*) and of a point (*c*) on the wire may be observed (¶ 262) accurately by a cathetometer. The increase of distance between *b* and *c* is evidently equal to the elongation of *ac*. In the Physical Laboratory of Harvard University the effects due to the yielding of the support are avoided by keep-ing the same weight always upon it. The wires (which are nearly 6 metres long) are attached to a beam by means of a piece of iron (*abd*, Fig. 178, 3) shaped like an inverted T. At the middle of the T a split plug (*c*) driven upwards into a vertical hole firmly grasps the wire. Side wires from the arms of the T hold a small platform (*g*) just above the floor. The weights to be used in stretching this wire are kept on this platform when not in use. Obviously the beam and the stem of the T are subjected to the same strain whether the load be suspended from the central wire or by the side wires.

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[1] This device is due to Mr. Forbes, of the Roxbury Latin School.

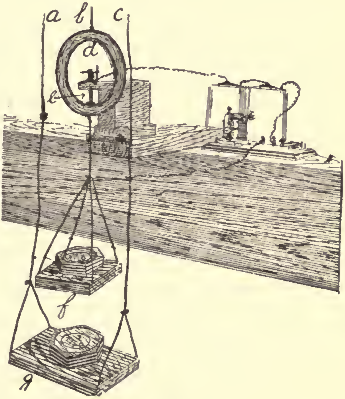

Fig. 179.

A stout ring (*de*, Fig. 179) is attached to the central wire (*b*) by a split plug (*d*). The stretching of the wire is measured by a micrometer, the point of which touches a small level surface on the ring at *e*. The contact is determined by electrical connections, as in ¶ 162. Directly below the point of contact a platform, *f*, is suspended, for the purpose of holding the weights by which the wire is to be stretched. There are many theoretical objections to this form of apparatus, which being of no practical importance have been left out of consideration. It is obviously necessary that the wire should be straight before the stretching forces are applied. For this purpose, a small load is always kept on it. In the apparatus shown in Fig. 179, the weight of the ring (*de*) and platform (*f*) should be sufficient to straighten the wire. In calculating Young's Modulus, we consider only the weight which must be *added to* the load already borne by the wire, in order to produce the observed elongation.

To determine the elongation in question, a reading of the micrometer must be taken with and without the weight. The difference in the readings gives, allowing for the pitch of the screw (see ¶ 52), the distance through which the wire has been stretched by the weight in question.

For a determination of Young's Modulus of Elasticity, a fine steel wire will answer. Care must be taken, however, not to bend the wire sharply over the edge of the vices or split plugs to which it is fastened. If the wire is 0.25 *mm.* in diameter, and free from kinks or bends, it may be made to bear safely a total load of 1, 2, or even 3 kilograms.

If *f* is the force exerted by the weight when reduced to dynes (see ¶ 166), e the resulting elongation of the wire in *cm.*, l the length in *cm.* of that portion of the wire in which the elongation takes place, and q its average cross-section in *sq. cm.*, Young's Modulus of Elasticity (E) is found in *C. G. S.* units (§ 8) by the formula

$$E = \frac{fl}{qe},$$

or by the method of reduction explained in ¶ 166.

EXPERIMENT LXVI.

BREAKING STRENGTH.

¶ 168. **Determination of the Breaking Strength of a Wire.** — A steel or spring-brass wire about $\frac{1}{4}$ *mm.* in diameter (No. 31, B. W. G.), free from kinks or sudden bends, is to be attached at one end to the eye (b, Fig. 180) by which the hook (bc) is attached to the spring balance (abc). The other end is to be fastened to some fixed point, as, for instance, a nail (e) driven into a post (d). A bobbin, c, is to be cut out (as shown in c' and c'' of the cross-sections 2 and 3), so as to fit over the hook of the balance without danger of turning. A few turns of the wire are made about the bobbin; the rest is wound around

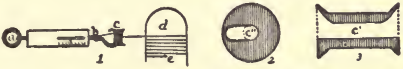

FIG. 180.

a post, *d*. The index of the balance is to be watched as a steadily increasing force is applied to the wire.[1] When the wire breaks, the maximum reading is recorded. The position of the break must now be ascertained. If it occurs at *b*, or between *b* and *c*, the result must be thrown out. If the wire breaks at *c* or at *d*, the accuracy of the result is doubtful; because a sharp bend in a wire where it passes round a corner may cause it to break under forces far less than its average breaking strength. If the break occurs between *c* and *d*, the break is probably a fair one. Enough wire will probably remain about the post for several repetitions of the experiment. The results should agree within five or ten per cent. Suspected results, much smaller than the average, may be discarded.

The cross-section of the wire must be found both by measurements with a micrometer gauge and by weighing a known length of the wire, let us say 1 metre, as accurately as possible. (See ¶ 166, formulæ I. and II.) The density of steel may be taken as 7.9, of brass 8.4 in this reduction. The student should compute by simple proportion the force necessary to break a wire one *sq. cm.* in cross-section; he may also calculate what length of the given wire would break under its own weight. Thus if 100 *cm.* of brass weighs 0.42 grams, its cross-section must be 0.42 ÷ 100 ÷ 8.4, or .0005 *sq. cm.* If it takes 2.94 kilograms to break such a wire, a wire 1 *sq. cm.* in cross-section would require 2.94 ÷ .0005 or 5,880 kilograms to break it. At 0.42 grams per metre, it would take 2.94 ÷ 0.42 or 7000 metres of the wire to break under its own weight.

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[1] *The hand should be held in such a position as not to be injured by the hook when the spring recoils.*

Obviously the result of this calculation should be the same whether a large or a fine wire is used, provided that the quality be the same, because both the breaking strength and the weight of a wire increase in proportion to its cross-section.

EXPERIMENT LXVII.

SURFACE TENSION.

¶ 169. **Determination of the Surface Tension of a Liquid.** — I. A piece of fine iron wire is bent as in Fig. 181, so as to form a fork (*fbg*) with parallel prongs (*cf* and *eg*) about 2 *cm.* apart. The fork is then suspended from the hook of a balance (*a*) so as to dip into a beaker of water, as in the hydrostatic method (Exp. 9). The fork must be entirely covered by water when the balance beam is lowered see (¶ 19); but when the latter is raised, the prongs only must dip into the water.

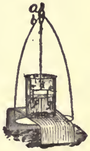

FIG. 181.

The weight of the fork is first balanced as accurately as possible; then the fork is lowered into the water, and suddenly raised out of it. A film of water will probably be found to fill the space between *fcdeg* and the surface of the water. This film will tend to pull the fork back into the water. To balance the pull which it exerts, an additional weight of about 3 decigrams must be placed in the opposite scale-pan. This weight is to be adjusted, by a number of trials, as accurately as possible. As the film gradually evaporates, it becomes lighter and lighter; but as its weight is, in any case, so small that it may be neglected, the change of weight will probably have no visible effect. The student will notice that the tension of the film of water remains sensibly constant as it grows thinner and thinner, until it breaks. This is entirely unlike the tension of solid substances, which depends upon their cross-section. The tension which liquids exert depends simply upon the *breadth of the surface* which tends to contract, not on the cross-section of the solid contents included by that surface. For this reason, the phenomenon is called "surface tension."

In the case under consideration, the film has two surfaces, each let us say 2 *cm.* broad. The total breadth of surface is therefore 4 *cm.* The student is to calculate what force (in dynes) is exerted by a *single* surface 1 *cm.* broad.

The surface tension of liquids depends upon temperature; hence the temperature should be noted. It is greatly affected by impurities in the liquids. An invisible quantity of oil, for instance, produces variations of ten or twenty per cent. Great care must therefore be employed in obtaining the purest distilled water. Both the inside of the beaker and the lower part of the wire should be cleaned with caustic potash, and afterwards rinsed in several changes of distilled water. The parts thus cleaned must not afterwards be touched by the finger.

II. A piece of thermometer tubing with a round bore about $\frac{1}{4}$ to $\frac{1}{2}$ *mm.* in diameter is carefully cleaned with caustic potash, which may be sucked through it with a medicine dropper (of course not by the mouth), then cleaned with distilled water. It is now dried by heat and filled with mercury. The contents are to be placed in a beaker, and weighed. If the quantity of mercury is too small to be weighed accurately, ten tubefuls may be weighed together (§ 39). The length of the tube is to be measured. The tube is now placed in a clean beaker containing pure distilled water (see I.). It should be at first inclined somewhat, so that the water which rises into it through "capillary attraction" may thoroughly wet its inside surface. It is next made vertical (see Fig. 182). The height of the column of water in the tube above the level in the beaker is then measured, both when it barely dips into the water, and when it dips so deep that the water rises nearly (but not quite) to the top of the tube. Other measurements should be taken similarly with the tube turned end for end. All results should agree closely, if the tube is of uniform calibre.

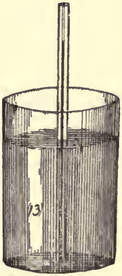

FIG. 182.

¶ 170. **Calculations relating to Capillary Attraction.** — If w is the weight in grams of the mercury which fills a tube, 13.6 the density of the mercury, and l

the length of the tube in *cm.*, the cross-section is (see ¶ 166, formula II.)

$$q = \frac{w}{13.6\,l}.$$

The radius of the tube is connected with the cross-section by the formula

$$q = \pi r^2;$$

hence, solving, we find

$$r = \sqrt{\frac{q}{\pi}} = 0.564\sqrt{q}, \text{ nearly.}$$

If h is the average height of the water in the tube above its level in the beaker, 1.00 the density of water, the volume of water raised is qh, or $\pi r^2 h$; the weight in grams is $1.00 \times qh$, or $1.00 \times \pi r^2 h$, and the weight in dynes (allowing g dynes to the gram) is qhg, or $\pi g r^2 h$. This weight, neglecting the buoyancy of the atmosphere, is sustained by the tension of a film lining the inside of the tube. The breadth of this film is evidently equal to the circumference of the tube ($2\pi r$). If a film $2\pi r$ centimetres broad can sustain a force $\pi g r^2 h$ dynes, a film 1 *cm.* broad would evidently sustain $\pi g r^2 h \div 2\pi r$, or $\frac{1}{2}\,grh$ dynes. That is, the "surface tension" of water (S) is given by the formula

$$S = \tfrac{1}{2}\,grh = 490\;rh \textit{ dynes per centimetre (nearly).}$$

Obviously, if S is constant, the product, $r \times h$, must be constant; that is, the height to which a liquid will rise in a tube is inversely as the radius of that tube.

EXPERIMENT LXVIII.

COEFFICIENT OF FRICTION.

¶ 171. **Determination of Coefficients of Friction.** — I. A piece of planed plank (*b*, Fig. 183) measuring let us say 5 × 20 × 40 *cm.*, is drawn horizontally by a spring balance, *a*, over a planed board *c*. The force necessary to maintain a uniform velocity after the plank is once started, is observed and noted. Then the plank is suspended from the spring balance and weighed. The ratio of the force required to draw a body to the force required to lift it is called a "coefficient of friction." The coefficient of friction in this case is that of wood on wood. If the force of traction varies in different parts of the board, the average should be calculated; and from this the average coefficient of friction may be found. It is instructive to repeat the experiment with the plank edgewise, so as to see whether the diminished area of the surfaces in contact is or is not compensated for by the increased intensity of pressure. For a fair comparison, the side and the edge of the plank

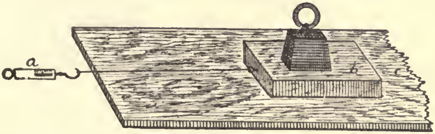

FIG. 183.

should of course be equally smooth, and both parallel to the grain of the wood.

The experiment may also be repeated with the plank flatwise, but with a heavy weight upon it as in the figure. The value of this weight should be found as in ¶ 159, and added to that of the board, in calculating the coefficient of friction in question.

The student will notice that it takes considerably more force to start a body than to drag it after it is once started. This is attributed to the cohesion of particles which takes place at various points, particularly when two surfaces remain long in contact. The ratio of the force required to start a body when resting upon a horizontal surface to the force required to lift it is sometimes called the "coefficient of starting friction." This must not be confounded with the ordinary "coefficient of friction."

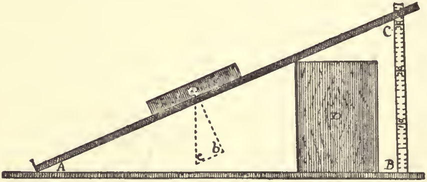

Fig. 184.

II. The board *A C* (Fig. 184) already used in I. is inclined (by means of a nail, *A*, and a block *D*) so that the plank *a*, when once started, slides down it with uniform velocity. A measuring rod *BC* is placed at a point *B*, 1 metre from *A*, and the vertical distance BC to the under side of the board is then measured. The "slope" of the under surface ($BC \div AC$) is thus found. The slope necessary to maintain a uniform velocity may not be the same from one end of the board to the other. If it is not the same, the average slope should be calculated.

If we resolve the weight of the block ac into two forces, one, ab, perpendicular to the board AC, the other, bc, parallel to it, then by definition (see I.) the coefficient of friction is $bc \div ab$; but, by similar triangles, this is equal to the ratio of BC to AB, which measures the "slope" of the board AC. The average slope which must be given to this board in order that the plank, when once started, may slide down it with uniform velocity, gives accordingly the "coefficient of friction" between the two surfaces in contact. The result should agree closely with that determined as in I.

¶ 172. **Fluid Friction.** — When a well-shaped boat moves through water with a velocity of *v cm. per sec.*, the opposing force (F) which it encounters is approximately equal to the square of this velocity multiplied by the area (a) of the surface wet by the water, measured in *sq. cm.*, and by a certain constant, f (about ·003), which is called the *coefficient of friction of water*, that is: —

$$F = fav^2 \textit{ dynes}.$$

Coefficients of fluid friction must not be confounded with coefficients of friction in the case of solids, which are calculated in an entirely different way. The frictional resistance between two solid surfaces depends, as we have seen (¶ 171), upon the pressure between them, but not upon the relative velocity of the surfaces. On the other hand, the resistance offered by a fluid to the motion of a solid does not depend upon the pressure between the surfaces in contact, but does depend upon their relative velocity. The nature of the fluid, the shape and smoothness of the solid, modify the result; but the material of which the solid is composed is generally unimportant. The resistance offered by fluids to the motion of solids or the reverse depends upon disturbances which are wholly confined to the fluid. Every fluid has, therefore, its own coefficient of friction.

When a current of water flows through a *large*[1] tube of the length l and radius r (both in *cm.*), since the area of wetted surface is $2\pi rl$, the force opposing the flow is

$$F = 2\pi rlfv^2 \text{ (dynes).} \qquad (1)$$

This force is supplied by the pressure (p) of the water (measured in dynes per *sq. cm.*) exerted upon an area equal to the cross-section (πr^2) of the tube; that is: —

$$F = \pi r^2 p. \qquad (2)$$

Equating (1) and (2), we find, —

$$p = \frac{2lfv^2}{r} \quad (3), \quad \text{or } f = \frac{pr}{2lv^2} \qquad (4)$$

---

[1] In capillary tubes, the force encountered is proportional directly to the velocity (see ¶ 250). In tubes from 1 to 5 *mm.* in diameter, for velocities between 10 and 100 *cm. per sec.*, no simple law can be given.

The velocity (v) can be estimated from the cross-section of the tube and from the volume of water which flows through it in a given length of time (¶ 147, 4); the pressure may be found by a pressure-gauge (see Exp. 69) at the point where the water enters the tube, provided that there is a free outlet at the other end, and that both ends of the tube are on the same level. If, as in Fig. 185, one end is higher than the other by an amount ac, equal let us say to h, then if g is the acceleration of gravity and 1.00 the density of water, the hydrostatic pressure is (see § 63)

$$p = 1.00\,gh, \text{ nearly.} \qquad (5)$$

The length (l) of the tube may be directly measured. The capacity (c) may be found by measuring, or (as in ¶ 32), by weighing the quantity of water required to fill it. The cross-section (q) may then be calculated by the equation -

$$q = \frac{c}{l} \qquad (6)$$

Hence the radius (r) is given by the formula —

$$r = \sqrt{\frac{q}{\pi}} = \sqrt{\frac{c}{\pi l}} \qquad (7)$$

The coefficient of friction, f, may now be calculated by formula (4), since all the quantities are known.

The "resistance" of a tube to the flow of a given liquid may be defined as the pressure in *dynes per sq. cm.* required to maintain through that tube a flow of 1 *cu. cm. per sec.* Thus if a rubber tube (*ab*, Fig. 185) 2 metres long and 3 *mm.* in diameter is used as a siphon to conduct water from a cistern, *a*, to a point *b*, it will be found that the outlet (*b*) must be about 10 *cm.* below the level (*a*) in the cistern in order that water may flow through *ab* at the rate of 1 *cu. cm. per sec.* The hydrostatic pressure corresponding to a difference of level of 10 *cm.* is nearly 10 grams per *sq. cm.*, that is, 9800 *dynes per sq. cm.* The "resistance" is therefore about 9800 units.

The resistance of a conduit may also be defined as the power (in ergs per second) necessary to maintain a unit current (1 *cu. cm. per sec.*) through the conduit in question. This definition bears a strong resemblance to the definition of electrical resistance (§ 136). The fact that power is required to maintain a current through the tubes and valves of a water-motor, together with the friction between the solid parts of the motor, will be found to modify the "efficiency" of the machine. The next experiment relates to determinations of "efficiency."

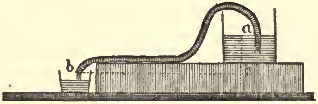

Fig. 185.

EXPERIMENT LXIX.

EFFICIENCY.

¶ 173. **Nature of Efficiency.** — Let us suppose that a 20-kilogram weight is suspended by a tackle (Fig. 186) consisting of two double blocks, with four cords passing between them. Let us first suppose that the cords run with absolute freedom round the pulleys which the blocks contain. The force on each cord must evidently be 5 kilograms; and a force of 5 kilograms, applied by a spring balance to the free end of the cord, as in the figure, will just hold the weight in place. If the weight were started upward by any impulse, no matter how small, the force of 5 kilograms constantly applied to the free end of the cord would (in the absence of friction) continue to raise it with a uniform velocity, until the two blocks met together. If the two blocks were 1 metre apart in the beginning, we should have 20 kilograms raised by the tackle through a height of 1 metre. Each of the four cords would be shortened 1 metre in this process, hence there would be 4 metres of slack to be taken up at the free end of the cord. The spring balance must accordingly retreat 4 metres. The work spent upon the machine by a force of 5 kilograms re-

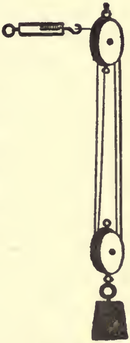

Fig. 186.

treating 4 metres (20 kilogram-metres), would be the same as that utilized by the machine in raising 20 kilograms 1 metre high (see § 14).

Let us now suppose that a slight downward impulse is given to the weight, so that it descends to its original position. The work spent by gravity upon the machine, being 20 kilogram-metres as before, is utilized in pulling the spring balance forward through a distance of 4 metres. In the absence of friction, the pull would be 5 kilograms as before. The amount of work utilized (20 kilogram-metres) would be equal, accordingly, to the amount spent upon the machine.

It is not necessary to consider the magnitude of the impulse by which the weight is started upward or downward; for if the weight moves with uniform velocity, it is capable of giving back this impulse, when it has been raised or lowered to any desired point (see § 121), in the act of stopping, when its energy of motion is lost. In the absence of all friction in the pulley-wheels, stiffness in the cords, and resistance in the air, a tackle devoid of weight would constitute a theoretically perfect machine, — that is, all the work spent upon it would be utilized by it. In practice, a considerable part of the work spent upon a machine is always transformed by friction into heat. That *proportion* of the work spent upon a machine which is utilized by it is called the "efficiency" of the machine.

Let us suppose that, instead of 5 kilograms, a force of 10 kilograms is required to raise a 20 kilogram weight by means of the tackle represented in Fig. 186. Then since, in raising 20 kilograms 1 metre, 10 kilograms retreat 4 metres, the work spent is 40 kilogram-metres; but the work utilized is only 20 kilogram-metres. The "efficiency" of the tackle as a machine for raising weights is accordingly $\frac{20}{40}$ or 50%.

Again, let us suppose that a weight of 20 kilograms, descending one metre, exerts a force of only 2 kilograms on the spring balance, which advances 4 metres. Then the work spent by gravity is 20 kilogram-metres, but that utilized is only 8 kilogram-metres; hence the efficiency of the tackle as a machine for utilizing potential energy (§ 122) is $\frac{8}{20}$ or 40%.

Finally, let us consider the tackle as a machine for storing and utilizing energy. A force of 10 kilograms is required to raise the weight, and this force must retreat 4 metres to raise the weight 1 metre. 40 kilogram-metres of work are thus spent upon the machine. The free end of the cord is now attached to some resistance which it is desired to overcome. A force of 2 kilograms is thus applied through a distance of 4 metres. The work utilized by the machine is only eight kilogram-metres. Evidently the efficiency of the tackle as a machine for storing and utilizing energy is only $\frac{8}{40}$ or 20%.

When energy is stored in a machine, part of it is lost. When this energy is utilized, part of what is left is lost. When energy undergoes a series of transformations, a certain proportion is lost in each.

Obviously, in stating the efficiency of a machine, it is necessary to specify where or how the work is spent upon it, and where or how the work is utilized.

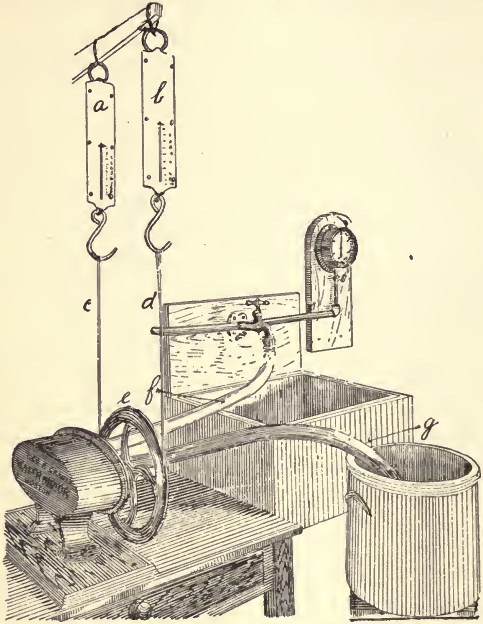

Fig. 187.

¶ 174. **Determination of the Efficiency of a Water-Motor.** — (1) To find the work *utilized by* a water-motor, the circumference of the driving-wheel (*e*, Fig. 187) is first measured, then two spring balances, *a* and *b*, are connected by a cord (*cd*) passing round the wheel. The motor is then started, and the tension of this cord increased until, through the friction which it exerts upon the wheel, the velocity of the latter is reduced to about one-half of its maximum. The speed of the wheel is then determined by counting the number of revolutions made in a given length of time. The reading of each spring balance is also found. If it varies, several observations must be made, and the mean calculated.

The difference between the two readings is equal to the force opposed by friction to the motion of the rim of the wheel, and must be reduced to dynes or megadynes. If the value of this force in dynes is F, if the number of revolutions in one second is n, and if c is the circumference of the wheel in centimetres, then in traversing the distance cn centimetres against the force F dynes, the work done must be cnF ergs. If we suppose that the force reduced to megadynes is equal to f, then cnf represents the work in megergs. Since cnf megergs of work are performed against friction in 1 second, and might be utilized for turning machinery (see ¶ 175), we infer that the work thus utilized would be cnf megergs per second. This measures, therefore, the power of the machine.

(2) To find the work spent in *driving* the motor, we must measure the quantity of water which passes through it in a given length of time. The water may be collected in a stone jar (*g*, Fig. 187), and weighed on a pair of rough platform-scales (Fig. 188). The

pressure of the water must also be found by means of a pressure-gauge connected with the supply pipe (see Fig. 187). The gauge should be as nearly as possible on a level with the outlet by which water escapes from the motor. The pressure must be reduced to dynes (or megadynes) per square centimetre. If v is the calculated volume in cubic centimetres of the water which flows through the motor in one second, and if P is the pressure of this water in dynes per square centimetre, then the work spent on the motor is vP ergs per second (see § 118). If p is the value of this pressure when reduced to megadynes per square centimetre,[1] then the work spent on the machine is vp megergs per second.

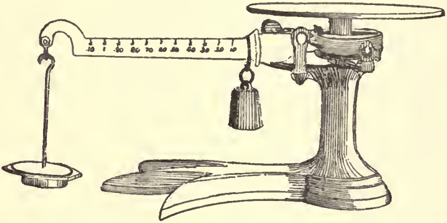

Fig. 188.

(3) For the accurate determination of efficiency, it is desirable to make *simultaneous* determinations of the power utilized by the motor, and of the power spent upon the motor. For this purpose, it is well for several students to work together. One may, for in-

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[1] The ordinary atmospheric pressure (15 lbs. per sq. in.) is equal very nearly to 1 megadyne per square centimetre. See Table 50.

stance, record the readings of the spring balance, *a*, another those of *b;* a third those of the pressure-gauge ; a fourth may attend to turning the stream of water into the stone jar at a given time, and cutting it off at a given time ; and a fifth may count the number of revolutions made by the wheel of the motor in the interval in question. When the experiment is performed by a single person, the mean readings of the balances and pressure-gauge must be inferred from observations just before and just after the determinations of velocity.

To calculate the efficiency (e) of the motor, the work *utilized* in one second *by the machine* is to be divided by the work *spent* in one second *on the machine*. We have, accordingly, —

$$e = \frac{cnf}{vp}.$$

In repeating the experiment, the tension of the cord should be increased or diminished. The maximum *power* of a water-motor is usually realized when, by the resistance which it has to overcome, the speed of the motor is reduced to about half its maximum speed. To obtain the maximum *efficiency*, the speed of the motor must be still further reduced.

¶ 175. **The Transmission Dynamometer.** — To measure the power of a motor actually doing useful work, a transmission dynamometer must be employed. One of the simplest forms of this instrument is represented in Fig. 189. Instead of carrying two cords (*c* and *d*) from the driving-wheel (*g*) of the motor to two spring balances (a and b) as in Fig. 187, these cords are made to pass around two pulleys (a and b, Fig. 189) to a second wheel (h), to which the motion is thus transmitted. The pulleys are suspended by two spring balances (A and B). The work done by the motor depends as before upon the difference in tension of the cords c and d; but if the pulleys run freely, the tension of e and f will be the same as that of c and d respectively; hence the forces A and B registered by the spring balances A and B (allowing for the weight of the pulleys) will be $2c$ and $2d$, respectively. It follows that $(c-d) = \frac{1}{2}(A-B)$. The difference between the readings (A and B) must therefore be halved in order to find the difference of tension between the cords[1] c and d.

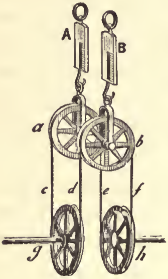

Fig. 189.

When the wheels move so fast that the revolutions cannot be counted, we may find the velocity of the cord, *cdef*, by measuring its length and counting the successive returns of a knot in the cord taking place in a given length of time. In other respects the work utilized is calculated as in ¶ 174, 1.

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[1] In practice, if the cord c is approaching g the tension on c will be a little greater than on e; and the tension on d will be a little less than on f, hence the difference of tension between c and d will be greater than the difference between e and f. That is, the work done by g will be a little greater than that received by h. The average between these two quantities is measured by the dynamometer.

EXPERIMENT LXX.

MECHANICAL EQUIVALENTS.

¶ 176. **Different Methods for determining the Mechanical Equivalent of one Unit of Heat.** — (1) If a weight (*d*, Fig. 190) is suspended by a cord passing over a pulley (*a*) and round an axle (*c*), surrounded with water in a calorimeter, and made to descend slowly to a position *d′*, by applying a suitable resistance through a friction-brake, *b*, the work done by gravity in pulling the weight, let us say *w*, through the distance *l* (equal to *dd′*) will nearly all be converted by friction into heat within the calorimeter. Let us suppose that the total thermal capacity of the calorimeter and its contents is *c*, and that its rise in temperature is $t°$; then the quantity of heat developed is *ct*. If gravity exerts a force of *g* dynes on one gram, it will exert *wg* dynes on *w* grams; and a force of *wg* dynes acting through the distance *l*, must perform a quantity of work equal to *wgl* ergs (§ 14). If *wgl* ergs are equivalent to *ct* units of heat (§ 16), one unit of heat must be equivalent to $wgl \div ct$ ergs. To obtain exact results, allowances must be made for the friction of the pulley, *a*, for loss of heat by cooling, etc. By a device similar in principle to the one described above, Joule

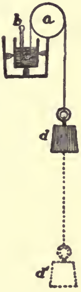

Fig. 190.

found that the mechanical equivalent of one unit of heat is about 41,660,000 ergs.

(2) Two heavy iron bars, *A* and *B*, suspended as shown in Fig. 191, may be released simultaneously by burning a cord (see ¶ 148) or by electrical means, so that when the bars meet endwise, a lead bullet (*b*) may be crushed between them. The work done by gravity in giving velocity to the bars is thus nearly all transformed into heat, through friction of the particles of lead against one another. Most of the heat will accordingly be found in the bullet. If the bullet is immediately lowered into a small calorimeter (*c*), the quantity of heat may be measured in the ordinary way (see ¶ 92). To obtain exact results, an allowance must be made for the energy of motion which remains in the bars after impact. If *l* is the *difference* between the original height of the bars and the height attained by them in their rebound, and *w* their combined weight, the work done by gravity is, as in (1), *wgl*. There is no way of allowing accurately for the energy taken up by the bars in the form of vibration, or for the energy of motion directly con-

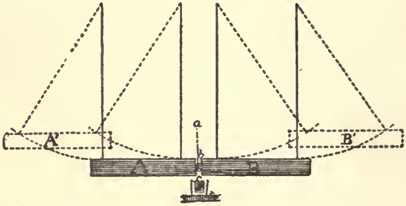

Fig. 191.

verted within the bars into heat. It is said that the proportion of energy thus lost is small.[1]

(3) By measuring the temperature of a water-fall above and below the fall, it would be possible to estimate the mechanical equivalent of heat. Thus if the water is 0°.1 warmer at the foot of Niagara Falls than above the falls, where the height is 42.5 metres, we should infer that to cause a difference of 1°, a water-fall must be 425 metres high. Each gram of water falling 425 metres, or 42,500 *cm.* under a force of 980 dynes, nearly, must receive from gravity 980 × 42,500, or nearly 41,660,000 ergs, in the form of energy of motion. If the conversion of this energy into heat warms it 1°, then the mechanical equivalent of 1 unit of heat must be 41,660,000 ergs.

In practice, the difference of temperature between the top and bottom of a water-fall is generally too slight to be measured accurately with ordinary instruments. Unless, moreover, the volume of a water-fall is very great, evaporation and other causes may affect the result. A rough experiment illustrating this method of determining mechanical equivalents will be described in the next section.

¶ 177. **Determination of Specific Heats by Mechanical Equivalents.** — A kilogram of lead shot is placed in a pasteboard tube (*ac*, Fig. 192) about 5 *cm.* in diameter and 120 *cm.* long, closed by two corks, *a* and *c*.

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[1] For an experiment similar in principle, performed by Hirn, see Trowbridge's New Physics, Exp. 105. This modification of Hirn's method is due to Professor Guthrie. The geometrical principles connecting arcs and heights have been already considered in the case of a ballistic pendulum (see § 109).

The free space between the cork, *a*, and the level of the shot, *b*, is to be measured with a metre rod. The cork (*a*) must be removed for this purpose, and its thickness allowed for. A thermometer is now fitted through the cork (*a′*, Fig. 193) so that by inclining the tube the bulb may be completely surrounded by the shot. The temperature of the shot is to be taken; then the thermometer is removed and the hole closed by a wooden plug. The tube is now inverted 100 times in rapid succession. During each inversion the centre of the tube is held at a fixed height. The shot are kept at one end of the tube by centrifugal force until this end comes vertically over the other. Then the rotation should cease, so that the shot may fall through the distance *ab* almost like a solid mass. Care must be taken, however, not to heat the shot through agitation which would result from too suddenly arresting the motion of the tube. The cork, *c*, should be supported by a table or other solid object so as not to yield under the blow given to it by the shot. Under this condition only, the energy of motion of the shot will be converted into heat *within the mass of shot.* The temperature of the shot is again observed in the same manner as before. It should have risen 5 or 6 degrees.

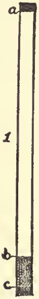

Fig. 192.

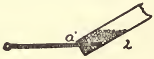

Fig. 193.

The experiment is now to be repeated with 1 kilogram of a substance in the form of shot, but of unknown specific heat; for instance, an alloy of zinc and lead. If this substance takes up more space than the lead, the distance fallen through in each reversal of the tube will not be quite so great. In this case more than 100 reversals may be made. The total distance fallen through should be as nearly as possible the same. Thus, if the distance *ab* is 100 *cm.* in the case of the lead shot, and 98 *cm.* in the case of the alloy, the tube should be reversed 102 times in the latter case, instead of 100 times.

¶ 178. **Calculations relating to Mechanical Equivalents.** — If s is the specific heat of the lead shot, w its weight in grams, g the weight of 1 gram in dynes, d the distance in *cm.* fallen through in each reversal, n the number of reversals, and J the mechanical equivalent of 1 unit of heat, then the total work done by gravity is evidently $wg \times nd$ ergs; and the heat into which it is converted is (neglecting all corrections) wst units, which is equivalent to $Jwst$ ergs. We have, therefore, —

$$Jwst = wgnd;$$

whence

$$J = \frac{ndg}{st}.$$

It is interesting to compare the value of J calculated by this formula with that found by Joule (see ¶ 176, 1). On account of many large corrections which have not been considered, the result will probably be too great by some 20 or 30 per cent. The principal source of error usually lies in the cooling of the shot by contact with the sides of the pasteboard tube. This can be avoided by cooling the shot before the experiment to a temperature about 6° below that of the tube. Before repeating the experiment, the tube must be allowed to return to its original temperature. The remaining errors have been found in the long run to balance one another with a probable resultant of about 10 per cent., which may be positive or negative according to the manner in which the manipulations are performed. Instead of computing the mechanical equivalent of heat, we may calculate the specific heat of the lead shot by the formula —

$$s = \frac{ndg}{Jt},$$

where J may be taken as 41,660,000; and if we distinguish by a prime (′) the qualities of an unknown substance, we find similarly, —

$$s' = \frac{n'd'g}{Jt'}.$$

Dividing, we find

$$\frac{s'}{s} = \frac{n'd't}{ndt'}, \text{ or } s' = \frac{sn'd't}{ndt'}.$$

In other words, the specific heats of two substances are to each other as the distances through which they must severally fall in order that each may be raised 1° in temperature. On account of the manner in which the two experiments are performed, the values of s and s' should be affected by constant errors in the same proportion, and hence the ratio between them will be affected only by accidental errors (§ 24). The last formula is therefore less inaccurate than the preceding formulae. To obtain the most accurate results by the aid of mechanical equivalents, as has been described, special devices should be employed to limit the fall of the shot to a given distance. In the absence of due precautions in this respect, the results must be expected to compare unfavorably with those obtained by the ordinary methods (see Exps. 33 and 34). It is nevertheless considered desirable that a student should familiarize himself with a definite example of the conversion of work into heat.

MAGNETIC MEASUREMENTS.

EXPERIMENT LXXI.

MAGNETIC POLES.

¶ 179. **Determination of the Distance between the Poles of a Magnet.** — Compound magnets composed of thin strips of steel bolted together will be found convenient for several experiments in magnetism. Such a magnet, formed of pieces of clockspring, 10 or 15*cm.* long, and 1 or 2*cm.* broad, is represented in Fig. 194. In fitting the strips together it may be necessary to soften them by heat; but their temper must be restored (by again heating and *suddenly* cooling them) before they can be thoroughly magnetized. Each strip should be magnetized separately by stroking one end of it ten times from the centre outward with or upon the south pole of a powerful electromagnet. This end will become a north pole (§ 126). The other end is then to be magnetized similarly by the north pole of the electromagnet. The strips are afterward bound together with all the north poles turned carefully in the same direction.

Fig. 194.

A piece of "ferroprussiate paper"[1] prepared for making "blue prints" is now to be stretched flat over a pane of window-glass, or over a stiff piece of pasteboard, with the sensitive surface uppermost. It is then to be placed over a powerful bar magnet constructed as has been described; and a few iron-filings are to be scattered over it. When the paper is jarred the iron-filings will arrange themselves as in Fig. 195. The sensitive surface is now to be exposed for about five minutes to direct sunlight, or to the light of the sky for a much longer period, until the surface not covered by the filings becomes quite

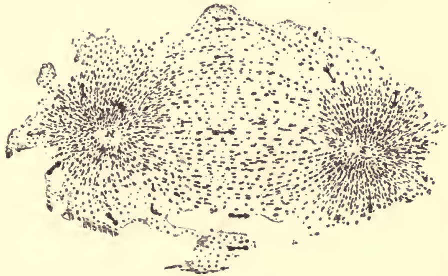

FIG. 195.

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[1] To prepare ferroprussiate paper, take 1 gram citrate of iron and ammonia, 1 gram *red* prussiate of potash, pulverize together and dissolve in 10 grams of water. This quantity should cover 20 or 30 square decimetres of smooth (not porous) paper. It should be applied by lamplight, as rapidly and evenly as possible, with a small sponge, in strokes first lengthwise then crosswise, then dried in the dark. The student is cautioned that all "prussiates" are poisonous. Ferroprussiate paper, already prepared, may be bought of dealers in photographic apparatus.

blue. It is then to be placed in the shade, and the iron-filings removed.

The surface covered by the iron-filings should not have been affected by the light; hence the arrangement shown in Fig. 195 should be represented by a white tracing on a blue ground. To make the print permanent, it is necessary to soak it in water for about ten minutes, after which it may be dried in the sun. To avoid delay in waiting for the print to dry, the student is advised to defer this "fixing process" until the end of the experiment. In the meantime, the print should be protected from excessive light either from the sun or from the sky. It may be illuminated freely by lamplight or gaslight.

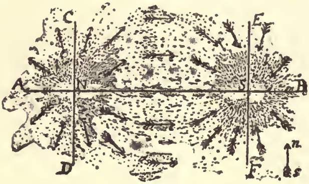

Fig. 196.

The magnet is now to be placed *over* the print, directly above its former position. A small compass with a needle not more than 1*cm.* long, is to be put beside the magnet at different points in the print. The direction of the north pole of the compass-needle is to be indicated in each case by an arrow drawn in pencil upon the paper (see Fig. 196). The direction of the arrows should agree closely with the lines of iron-filings, although the compass-needle is in a slightly different plane. The results of this experiment will be somewhat affected by the earth's magnetism. It is well, therefore, to note the direction (*sn*) in which the compass points when the magnet is removed to a distance.

A line *AB* is now drawn so as to bisect as nearly as possible the areas *N* and *S*, from which the "lines of force" (§ 127) seem to diverge. The line (*AB*) should agree with the general direction of the lines of force between *N* and *S*, whether indicated by the compass-needle or by the iron-filings. The areas *N* and *S* are again to be bisected by lines (*CD* and *EF*) perpendicular to *AB*. These lines should cut the edge of the areas (*N* and *S*) at a point where the lines of force are also perpendicular to *AB*.

The positions of the poles *N* and *S* are determined by the intersection of the first line (*AB*) with the perpendiculars (*CD* and *EF*.) The distance between the poles is to be measured. The experiment is to be repeated with at least two other magnets as nearly as possible like the first.

The student may be interested to make prints showing the arrangement of iron-filings due to two parallel magnets, both when their north poles are turned in the same direction and when turned in opposite directions.[1]

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[1] See Experiment 40 in the Elementary Physical Experiments published by Harvard University.

EXPERIMENT LXXII.

MAGNETIC FORCES.

¶ 180. **Determination of the Strength of Magnetic Poles.** — One of the magnets (*ef*, Fig. 197), used in Experiment 71, is now to be placed horizontally in the pan-holder (*c*) of a balance (the pan being removed), and counterpoised by an observed weight in the opposite pan (*a*). A second magnet (*gh*) is to be placed directly under the first, and parallel to it.

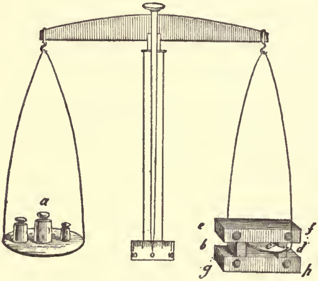

Fig. 197.

The north poles are at first to be turned in opposite directions, so that the magnets may attract each other. Small blocks (*b* and *d*) are now placed between them to keep them apart. The thickness of the blocks should be such that when the balance beam is raised upon its knife-edges, the index (*b*) may point to zero. The weight in the pan *a* is then gradually increased until the magnets are pulled apart. Care must be taken to find the *greatest* weight which the magnets can sustain; for if they be once separated a much smaller weight can hold them apart. In the final adjustment small weights (not over 1*cg.*) should be let fall into the scale-pan from a height not exceeding 1*cm.* The weight necessary to pull the magnets apart is to be noted.

The magnet *gh* is now to be turned end for end, so as to repel *ef*, and the weight in the pan *a* is gradually to be diminished until the magnet *ef* just touches the blocks (*b* and *d*). When a small weight is added to the pan *a* the beam will not turn suddenly as in previous observations; but, being in stable equilibrium, it may balance in any position. Care must therefore be taken to find the *smallest weight* which can cause a separation of the magnets, however slight.

The mean distance between the magnets, from centre to centre, is now to be determined by measuring the thickness of the magnets and the thickness of the blocks with a vernier gauge. In setting the gauge upon a magnet, if the jaws are of iron or steel the blocks of wood (*b* and *d*) should be interposed between the jaws and the surfaces of the magnet, since the strength of the magnet might otherwise be perceptibly affected. The thickness of the blocks may then be found and allowed for. The experiment should be repeated with a third magnet, let us say *ij* in place of *gh*; then with *gh* in place of *ef*. In this way the forces of attraction and repulsion between each pair which can be formed out of the three magnets will be determined.

The student may be interested to prove that it makes no difference which of two magnets is the one suspended. This fact is an illustration of the general principle that action and reaction are equal and opposite. It will be noticed that the attraction between two magnets when close together, is much greater than their repulsion. This is due to the effects of induction (see § 129, footnote).

¶ 181. **Calculations relating to Magnetic Forces.** — If w be the weight in grams necessary to counterpoise a magnet; w_1 the weight of the counterpoise necessary to lift the magnet and at the same time to pull it away from the attraction of a parallel magnet at the distance d; and w_2 the weight similarly required when the two magnets repel each other; then if 1 gram $= g$ dynes, the force of repulsion which we call positive is $+ (wg - w_2 g)$ dynes, and the force of attraction, which we call negative, is $-(w_1 g - wg)$ dynes. The numerical sum, or algebraic difference, Δ, between these forces is accordingly $(w_1 g - w_2 g)$ dynes. Substituting this value in the formula of § 129, we have, if any two of the magnets are equal in respect to the strengths (s and s') of their poles,[1]

$$ss' = s^2 = \frac{\Delta\, d^2}{4};\ \text{ or } s = \frac{d}{2}\sqrt{(w_1 - w_2)\, g}.$$

Thus if the attraction between two nearly equal magnets at a distance of 2 *cm.* is 600 dynes, and the repulsion 300 dynes, a force of 900 dynes (0.92 *g.*, nearly) will be required to offset the effect of reversing one of the magnets. the mean strength of their poles is, accordingly, about $\frac{2}{2}\sqrt{.92 \times 980}$, or 30 units each.

The results of this experiment are subject to errors which are sometimes (though rarely) almost as great as the quantities measured. They are nevertheless valuable in enabling us to form an *immediate estimate* of the strength of magnetic poles, which, though rough, may guide us in the less direct but more accurate methods which follow.

---

[1] If no two of the magnets are equal, we must form three equations from observations made with each pair of magnets; thus —

$$ss' = \frac{\Delta\, d^2}{4}\ (1);\ ss'' = \frac{\Delta' d'^2}{4}\ (2);\ \text{and } s's'' = \frac{\Delta'' d''^2}{4}\ (3).$$

Multiplying (1) and (2) together and dividing by (3) we have —

$$s^2 = \frac{\Delta\,\Delta'}{4\,\Delta''} \times \frac{d^2\, d'^2}{d''^2};\ \text{or } s = \frac{dd'}{2\,d''}\sqrt{\frac{\Delta\,\Delta'}{\Delta''}}$$

EXPERIMENT LXXIII.

MAGNETIC MOMENTS.

¶ 182. **Determination of the Couple exerted by the Earth's Magnetism on a Suspended Magnet.** — A magnet (*gh*, Fig. 198) used in Experiment 72 is to be suspended horizontally by a wire *cf*. The coefficient of torsion of the wire has been found in Exp. 64. The wire is attached at *c* to a knitting-needle (*bd*) revolving on a graduated circle (*ae*) as in the torsion balance (Fig. 176, ¶ 165). The wire is, however, vertical, and the circle horizontal in this experiment. A short piece of wire should be attached vertically by wax to each end of the magnet to serve as a sight. The needle is first turned so that the north pole of the magnet points north, and its reading is taken. Then it is turned until the magnet points east, and the reading again taken. A distant object should now be sighted in the direction indicated by the sights. The needle is then turned so that the magnet points west. The same distant object should be in line with the sights. The reading of the needle is again observed. The experiment should be repeated with the other magnets employed in Experiment 72.

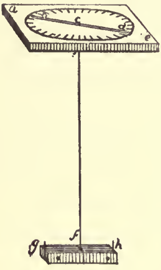

Fig. 198.

If the poles of the magnet are l centimetres apart, if they contain s units of magnetism each, and if the earth exerts on each unit of magnetism a force which has a horizontal component equal to H dynes, then the s units of magnetism in the north pole must be urged northward with a force of Hs dynes, and the south pole will be urged southward with an equal force. The two forces will constitute a couple (§ 113) C, with an arm equal to the distance l, between the poles; since the magnet is at right-angles to the forces in question. We have, therefore,

$$C = Hsl, \text{ or } H = \frac{C}{sl}.$$

This couple must be balanced by an equal and opposite couple due to torsion in the wire. It is obvious that in turning the magnet end for end it must be made to revolve through 180° so as to make an angle of 90° (on the average) with its original (north and south) direction. To produce torsion in the wire the needle must be turned through *more* than 180° in all, or more than 90° from its original setting.

Let us suppose that the needle has revolved through a total angle a, or an average angle of $\frac{1}{2}\, a$ from its original position; if the magnet had remained pointing to the north the twist in the wire would be $\frac{1}{2}\, a$; but the revolution of the magnet through 90° causes the wire to untwist through 90° at its lower end. The angle of torsion is therefore $\frac{1}{2}\, a - 90°$. It is now easy to calculate the couple exerted by the earth. If it requires a couple of t dyne-centimetres to twist the wire through 1° (see Experiment 64) it must require $(\frac{1}{2}a - 90) \times t$ dyne-centimetres to twist it through the angle in question. Substituting this value for c in the formula above we have —

$$H = \frac{(\frac{1}{2}a - 90)\, t}{sl}$$

It is interesting to estimate the value of H by the rough values of s and l already determined in Experiments 71 and 72. If, for instance, the distance between the poles is 10 *cm.*, and the strength of each 30 units, and if the couple produced is 50 dyne-centimetres, then the earth must exert a force of $\frac{1}{6}$ of a dyne on each unit of magnetism when free to move only in a horizontal plane. This is what is meant by the statement that the "horizontal intensity" of the earth's magnetism is $\frac{1}{6}$ or 0.17, nearly. In practice large errors would be committed in estimating the horizontal intensity in this way, on account of the uncertainty of the factor s (see ¶ 181). A much more exact method will be considered in connection with Experiment 74.

The student should note that the couples acting on suspended magnets are proportional to the products of the distance between the poles and the strength of the poles, both of which have been already determined. These products (sl, $s'l'$, $s''l''$) are called the *magnetic moments* of the magnets to which they respectively belong.

EXPERIMENT LXXIV.

MAGNETIC DEFLECTIONS.

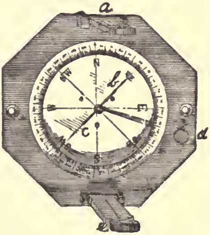

FIG. 199.

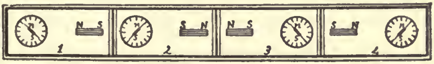

FIG. 200.

¶ 183. **Determination of Magnetic Deflections by means of a Magnetometer.** — A surveying-compass (Fig. 199) is placed in the middle of a wooden table, in the construction of which no iron has been employed even in the form of nails. All iron or steel objects are to be removed from the immediate neighborhood. The directions of the magnetic north, south, east, and west are to be determined by this compass, and marked by pencil lines upon the table. In all experiments in magnetism the magnetic points of the compass will be those referred to, unless otherwise stated. A magnet already tested in Experiment 71, considerably longer than the compass needle, is now placed at the east of the compass with its north pole toward the compass (see Fig. 200, 1). The distance of the magnet from the compass must be noted. It should be small enough to cause a measurable deflection of the compass, let us say 5 or

10 degrees, but at least twice the length of the magnet.[1] The position of each end of the magnet is then marked in pencil on the table, and the deflection of the compass observed by the reading of two pointers, attached one to each end of the needle.

The magnet is now turned end for end (as in Fig. 200, 2) and the deflection again observed. The experiment is to be repeated with the magnet at an equal distance from the compass, but at the west of it, as in Fig. 200, 3 and 4. There will thus be 8 readings in all, from which the average deflection of the needle may be calculated. The mean distance of the centre of the magnet from the centre of the needle may be found quite accurately by measuring the distance between the outer and between the inner pencil marks on opposite sides of the needle, adding, and dividing by 4. The experiment is to be repeated with the other magnets employed in Experiment 71.

The results of this experiment are to be reduced as will be explained in ¶ 185.

¶ 184. **Theory of the Magnetometer.** — When a magnet is placed near a compass-needle, and at the east or west of it, as in Fig. 200, so that one of its poles is nearer than the other, the needle is deflected under the influence of the nearer pole. The lines of force due to a magnet at any point nearly in line with the two poles are (see Fig. 195) nearly parallel to the magnet; and hence in the case which we have supposed they are nearly east and west. That is, the magnet tends to make the compass-needle point east and west.

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[1] For very accurate measurements the distance of the magnet from the compass should be at least 4 times the length of the magnet and 12 times the length of the needle.

Let us suppose that the magnet is at the east of the compass, and that its south pole is (as in Fig. 200, 2) nearer than the north pole. Then the north pole of the compass-needle (*c*, Fig. 201) will be attracted by the south pole of the magnet more than it is repelled by the north pole. The resultant force will therefore be an attraction toward the east, which we will represent by the line *cd* (Fig. 201). At the same time the earth pulls the north pole of the compass-needle northward, with a force represented let us say by the line *ca*. The resultant of these two pulls is a force *cb*, easily found by geometrical construction (§ 105).

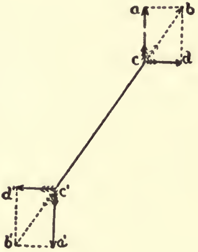

Fig. 201

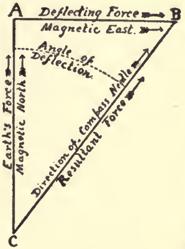

Fig. 202.

On the other hand the south pole of the compass-needle (*c'*) will be repelled by the south pole of the magnet more than it is attracted by the north pole. It will accordingly be urged westward with a force $c'd'$. At the same time it is drawn southward by the earth's magnetism with a force $c'a'$. The resultant force, $c'b'$, may be found as before. Assuming that the forces acting upon the south pole of the needle are equal and opposite to those acting upon the north pole, it follows that $c'b'$ must be equal and opposite to cb. If the needle cc' is free to turn, it will obviously take the direction of the two resultants.

The relation between the forces exerted by the earth and by the magnet upon the north pole of the compass-needle is shown in Fig. 202. The magnetic force is represented by AB; the earth's force by by CA; the resultant by CB. The angle BAC is called the angle of deflection. The tangent of this angle is by definition equal to $AB \div CA$; since AB and CA are at right-angles. Obviously, the magnitude of a deflecting force bears to that of a directive force at right-angles to it a ratio equal to the tangent of the angle of deflection produced.

It has been stated that when the two poles of a magnet are at unequal distances from a compass-needle, the nearer pole has the greater effect. Since the two poles are always equal and opposite, the action of a magnet as a whole evidently depends not only upon the strength of its poles, but also upon the difference of their distances from a given point. We must accordingly consider the length of a magnet, as well as the strength of its poles, in calculating the effect which it will produce. It is found, in fact, that the forces produced by different magnets at a given distance are very nearly proportional to the "moments" of the magnets in question, that is (see ¶ 182), to the products of the strength of the poles and the distance between them. The moments of the magnets (sl, $s'l'$, etc.) employed in this experiment have been already determined (¶ 182). If a, a', etc., are the deflections produced, we should have —

$$\frac{sl}{tan\ a}=\frac{s'\,l',}{tan\ a'}, \text{ etc., nearly.}$$

The student should satisfy himself that this is the case before proceeding to the calculations of the next section.

A compass, having on each side of it a pair of revolving supports, capable of holding several magnets, successively at a given distance from the needle, affords one of the most direct and accurate methods of comparing magnetic moments together, and is properly called a magnetometer.

¶ 185. **Calculations relating to Magnetic Deflections.** — EXAMPLE. Let us suppose that in Fig. 200 the average distance between the centre of the magnet NS and the centre of the needle ns is 25 *cm.*, and that the distance between the poles of the magnet (¶ 179) is 10 *cm.* so that as in (2) the south pole is 20 *cm.* from the needle and the north pole 30 *cm.* from it. Assuming that each pole has a strength of 30 units (see ¶ 181) the attraction of the south pole for a unit of positive magnetism *at the centre of the needle* (see § 129) must be $30 \div (20)^2$ or $\frac{3}{40}$ dyne. The opposite pole must exert a repulsion on the same unit of magnetism equal to $30 \div (30)^2$ or $\frac{1}{30}$ dyne. The resultant of these two forces is evidently $\frac{3}{40} - \frac{1}{30}$ or $\frac{1}{24}$ dyne acting in an easterly direction parallel to AB (Fig. 202). The earth's magnetism acts in a northerly direction parallel to CA (Fig. 202).

Now since $$\frac{AB}{CA} = tan\ CAB,$$

we have $$CA = \frac{ab}{tan\ CAB}$$

If, for example, $CAB = 14°$, the tangent of CAB is .249 (see Table 5) or $\frac{1}{4}$, nearly; then CA is evidently 4 times as great as AB; hence if $AB = \frac{1}{24}$ dyne per unit of magnetism, $CA = \frac{1}{6}$ dyne per unit of magnetism.

In practice an estimate of the earth's magnetism made in this way will be found to differ greatly from that made as in the last experiment, on account of a tendency to underestimate the strength of the magnetic poles in Experiment 71.

Let us suppose that this strength were estimated at 15 units instead of 30 units. Then in the calculation above we should have estimated the earth's field at $\frac{1}{12}$ dyne per unit of magnetism (instead of $\frac{1}{6}$). In ¶ 182, however, we should have estimated the earth's field at $\frac{1}{3}$ dyne per unit of magnetism. That is, our estimate in Experiment 73 would be too great, and that in Experiment 74 too small in proportion to the error originally made in estimating the strength of the poles. Now when one of two estimates is too great, and the other too small in a given proportion, the geometric mean between them must be equal to the quantity which we seek. Hence to find the true value of the horizontal component of the earth's magnetism, we multiply together the estimate of Experiments 73 and 74, and extract the square root of the result. Thus $\sqrt{\frac{1}{3} \times \frac{1}{12}} = \frac{1}{6}$. The result is independent of the value provisionally adopted for the strength of the magnetic poles. If the two estimates agree closely the arithmetic mean may be substituted for the geometric mean (§ 57).

Knowing now the true value of H, we may recalculate the moment (M) of the magnet and the strength of the poles by formulæ derived from ¶ 182:

$$M = sl = \frac{C}{H};\ s = \frac{C}{Hl}.$$

EXPERIMENT LXXV.

DISTRIBUTION OF MAGNETISM, I.

¶ 186. **Determination of the Distribution of Magnetism on a Rod by the Method of Vibrations.** — A steel rod (*aj*, Fig. 203) one metre long, and about 1 *cm.* in diameter, is marked with a file at ten points (*a* . . .*j*) 10 *cm.* apart, beginning with a point *a*, 5 *cm.* from one

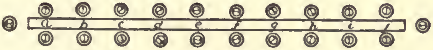

Fig. 203.

end of the rod. It is then magnetized by stroking it from *e* to *a* 10 times with the south pole of a powerful electro-magnet, and by stroking it 10 times from *f* to *j* with the north pole of this magnet. A small piece of a sewing-needle (*f*, Fig. 204) about 1 *cm.* long, and highly magnetized is attached horizontally by sealing-wax to a bullet *e*, and suspended by a fine fibre (*cd*) of untwisted silk from a cork (*a*) in a test tube (*bg*).

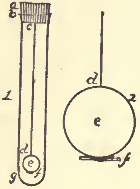

FIG. 204.

The torsion of the fibre (*cd*) should be so slight that the cork (*a*) may be twisted through 360 °, without deflecting the needle (*f*) more than a few degrees from the magnetic north, toward which one end should point. The needle is then to be deflected by a magnet; and when the magnet is suddenly taken away the needle should make a series of vibrations in a horizontal plane. The weight of the bullet should be so proportioned to the magnetic strength of the needle that there may be about 10 vibrations completed in one minute. The exact time required for 10 vibrations of the needle is to be determined when it is vibrating in an arc not exceeding 30 ° or 40° (see Table 3, *g*). The north pole of the needle should be distinctly marked.

The test tube is now to be placed opposite the end of the rod, then held successively on each side of each of the ten points (*a*—*j*, Fig. 203). The direction indicated by the north pole in each position is to be represented by arrows (drawn as in Fig. 203) the direction of which may be compared with that of the lines of force issuing close to the magnet in Fig. 196. In addition, the rate of vibration of the needle is to be determined by counting the number of vibrations completed in 1 minute, or in whatever time may have been required for 10 vibrations under the influence of the earth's magnetism alone. In all cases the arc of vibration should be limited to 30° or 40° (see Table 3, *g*).

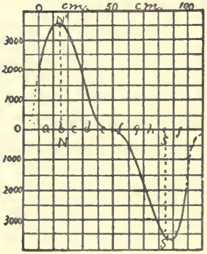

Fig. 205.

The number of vibrations made in the given time on one side of *a* is to be averaged with that made on the other side; and in the same way the average number of vibrations for each of the ten points is to be found. These numbers are then all to be squared (see Table 2). The results are to be plotted on co-ordinate paper (see § 59). Distances in centimetres are represented by a horizontal scale at the top of the figure, and the square of the number of oscillations is shown by the vertical scale at the left of the figure. Thus, if opposite the point *b*, 15 *cm.* from the end of the magnet, the needle makes 60 vibrations per minute, we place a cross at the right of the square of 60 (3600) and under 15 *cm.* The vertical distances are measured upward if the north pole of the needle is repelled by the bar, and downward if it is attracted by it. In the same way other points may be found through which a curve is to be drawn as in Fig. 205. Evidently, in this figure, *N* represents the "positive" or "north" end of the magnet.

This method of representing the distribution of magnetism depends upon the general principle that forces are proportional to the squares of the rates of oscillation which they produce (see § 110). The curve represents accordingly the strength of the magnet at different points as compared with the strength of the earth's magnetism. We should strictly allow for the effect of the earth on all the rates of oscillation; but as it is represented only by 100 units on the vertical scale, this effect would be hardly perceptible.[1]

The student should draw by the eye two vertical lines *NN′* and *SS′*, dividing each area enclosed by the curve as nearly as possible into two equal parts. The distance between these lines indicates approximately the distance between the poles of the magnets. This latter may therefore be found by the scale at the top of the paper.

EXPERIMENT LXXVI.

DISTRIBUTION OF MAGNETISM, II.

¶ 187. **Magneto-Electric Induction.** We have seen that when iron-filings are brought into the neighborhood of a powerful magnet, they tend to arrange themselves along certain lines called "lines of force." These lines of force are not, like the meridians upon the surface of the globe, purely geometrical conceptions. According to Tyndall, the apparently empty space between the poles of a powerful electro-magnet "cuts like cheese." The most surprising fact connected with this phenomenon is that a knife with which such a magnetic field is cut becomes temporarily electrified. The point and the handle of the knife resemble, for the time being, the two poles of a voltaic cell, from which a current of electricity can be derived by making the proper connections. It is not necessary to use a knife; any piece of metal, a wire for instance, will do as well. All tendency to produce a current ceases when the knife or wire stops moving, or as soon as all the lines of force have been cut. The effect of a sudden motion upon a galvanometer may accordingly be almost instantaneous. In such cases it is measured by the "throw" of the needle (§ 109). It is found that the "throw" is proportional, other things being equal, to the intensity and extent of that part of the magnetic field which has been cut through, or, according to a system of representation universally adopted, it is proportional to the *number of lines of force* which have been cut.

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[1] The effects of "induced magnetism" may introduce errors of 5 or 10 per cent in this experiment (see ¶ 207). The shape of the curve in Fig. 208 will not, however, be materially altered.

If a loop of wire is placed around the middle of a long bar-magnet (Fig. 206) and suddenly made to slip off one end of the magnet, it will evidently cut nearly all the lines of force on that end of the magnet.

A delicate galvanometer connected with the ends of the loop will be affected. This affords a convenient method of comparing the strengths of different magnetic poles. In practice we employ a coil of wire instead of a simple loop; for when each turn cuts all the lines of force, the effect is found to be proportional to the number of turns which the wire makes about the magnet. It is not necessary to slide the coil completely off the magnet. A motion of a few centimetres may affect the galvanometer. When the motion is confined to one end of the magnet it will be found to deflect the needle in opposite ways according to which way the coil is moved. In other words the direction of the electrical current depends upon the direction of the motion. Let us suppose the direction of the motion to be always the same, that is, from left to right, or from the north toward the south end of the magnet. Then the galvanometer will be deflected one way when the motion of the coil takes place near one end of the magnet, and the other way when it takes place near the other end of the magnet. That is, the direction of the electrical current depends on the direction of the lines of force. Near the middle of the magnet a neutral point will generally be found. If the coil be moved from this neutral point toward either end of the magnet, it follows from the statements made

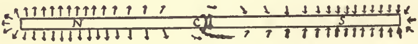

Fig. 206.

above that the direction of the current will always be the same. This direction is with the hands of a watch, as seen from the south pole of the magnet.

The throw of the needle is proportional, other things being equal, to the distance through which the coil is moved; hence it is important in comparing results that this distance should be always the same. If the coil is moved always through a given distance, the effect will be found to be greatest when the motion takes place near the ends of the magnet, where the lines of force are the thickest. In other words the magnitude of the electrical current depends upon the closeness of the lines of force. The effect is very nearly the same whether the coil moves *more* or *less swiftly*[1] through a given distance. In the first case we have a rapid motion, and hence a comparatively strong current lasting for a short time; in the second case we have a weaker current lasting for a proportionately long time. The forces exerted upon the galvanometer needle are proportional to the current; hence, by the fundamental law of motion (§ 106),

$$ft = mv,$$

since the product (ft) of the force and the time of its action is the same in both cases, the momentum given to the needle must be the same.

We shall make use of these facts to estimate the relative strength of the magnetism of a rod in different parts, and to distinguish positive from negative magnetism.

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[1] In order that this may be true, the duration of the motion must be several times less than the time occupied by one vibration of the galvanometer needle.

¶ 188. **Construction of an Astatic Galvanometer.** — A delicate galvanometer, such as has been already employed for the detection of currents created by a thermopile (Exp. 39), is represented in Fig. 207, and may be constructed as follows: —

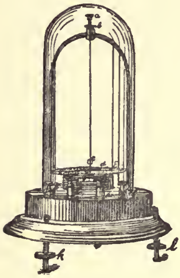

Fig. 207.

Two magnetized needles, *c* and *h* (Fig. 208), of nearly equal strength are connected by a vertical piece of wire, with their north poles *in opposite directions*, and suspended horizontally, by a fine thread (*bc*) of untwisted silk, from a screw *a*. This screw is held by a nut *b*, itself capable of rotation, so that the thread may be raised or twisted at pleasure. The two needles *c* and *h* should form a nearly "astatic" combination (*a* privative and ἵστημι, to stand); that is, one which, owing to the

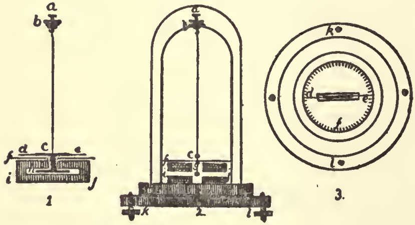

Fig. 208.

equal and opposite forces exerted upon it by the earth, has no strong tendency to stand in any particular position.

The strength of either magnet may generally be increased by stroking one of the poles, as in ¶ 179, with the dissimilar pole of a powerful magnet, or diminished by touching similar poles together. A very light touch is usually sufficient to produce a perceptible change in a magnet. The delicacy of the instrument depends upon the delicacy of the balance which can be established between the two needles. It is generally possible to make the combination point permanently east and west. In practice, however, the needles are magnetized so that the time occupied by one oscillation is 5 or 10 times as great as that of either needle by itself. The needle is then sufficiently astatic for most purposes. It may be remarked that the rate of oscillation of an astatic needle is the best test of its adjustment (see ¶ 193, 4).

100 metres of insulated copper wire about $\frac{1}{2}$ *mm.* in diameter are now to be wound on the two rectangular bobbins *f* and *i* (Fig. 208, 1 and 2).[1] The bobbins are shaped so that the lower needle (*h*) may hang inside of them, and the upper needle (*c*) just above

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[1] If it is desired to use the instrument later on (Exp. 86, II. and Exp. 95) as a differential galvanometer, the 100 metres of wire should be cut in two, and the two parts twisted together before winding them on the bobbins. The galvanometer will thus have four terminals instead of two. If two of the terminals are temporarily joined together, the other two may be connected with binding-posts in the ordinary manner.

them. Two indices of aluminum wire, *d* and *e* (Fig. 208, 1 and 3), are then attached to the upper needle, and a cardboard protractor (*f*) is set beneath them. The instrument is usually mounted on wooden supports, with levelling screws *k* and *l*, and covered with a glass shade to cut off currents of air. The galvanometer thus constructed should be sensitive to a few millionths of an ampère.

¶ 189. **Determination of the Distribution of Magnetism on a Rod by the Method of Induction.** — A coil (*b*, Fig. 209) consisting of about 100 turns of No. 20 insulated copper wire, wound on a brass bobbin, is fitted to a brass tube *ad* so as to slide freely between the stops *a* and *c*, through a distance of about 10 centimetres. The tube must be large enough to admit the long magnet employed in Experiment 75. It is first to be fastened near one end of this magnet by means of the clamp *d*, so that a point (*a*, Fig. 203) 5 *cm.* from the end of the magnet may come half-way between the stops *a* and *c* (Fig. 209).

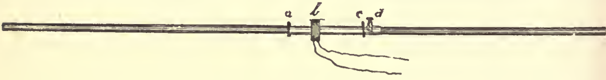

Fig. 209.

The needle of a delicate galvanometer (Fig. 207), such as has been already employed for the detection of electrical currents (Exp. 39), is now to be loaded, if necessary, by attaching small bits of lead with sealing-wax to each end of the needle, so that its time of oscillation may be at least 10 seconds. The instrument is to be set up with the plane of its coils approximately north and south. The nut *b* is then turned so that, by the torsion of the thread *bc*, the needle of the galvanometer is made to point to 0°. The terminals of the coil *b* (Fig. 209), are then to be connected with the terminals of the galvanometer.

The coil (*b*) is then suddenly made to slide from *a* to *c* (Fig. 209), and the throw of the galvanometer is noted. When the oscillation of the needle has ceased [1] the coil is made to slide back suddenly from *c* to *a*, and the throw of the galvanometer is again noted.

The experiment is to be repeated with the tube clamped so that other points (*b*, *c*, *d*, *e*, etc., Fig. 203) may come successively half-way between the stops *a* and *c* (Fig. 209).

In each case two throws of the galvanometer are to be observed. The direction of each throw is to be noted, and the average deflection calculated.

The positions of the centre of the tube with respect to the magnet are also to be noted. The results are to be plotted on co-ordinate paper as in Fig. 205,

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[1] The student should learn to stop the vibrations of a magnetic needle. If a magnet is directed toward a needle as in Fig. 200, ¶ 183, a deflection in either direction may be produced. If the magnet be turned so as to tend to cause a deflection at every instant opposite to the motions of the needle, the latter will come very quickly to rest. To stop a wide oscillation, the magnet must be brought near the needle, but when the oscillation becomes feeble, the process should be continued from a greater distance. To affect an ordinary astatic needle, the magnet should be held not only at right-angles with it, but also considerably above or below it. A perfectly astatic needle should not be affected by a magnet in the same horizontal plane.

¶ 186, except that the vertical distances are to represent throws[1] of the galvanometer needle, instead of squares of the rates of oscillation. If the throw in a given case is in the same direction as at the north end of the magnet when the coil is stopped in a given direction, the distances are to be measured upward; otherwise downward. From the curves thus obtained the poles of the magnet are to be located as in ¶ 186, and the distance between them is to be estimated. The result should agree closely with that obtained in the last experiment.

EXPERIMENT LXXVII.

MAGNETIC DIP.

¶ 190. **The Earth's Magnetism.** — If fine iron-filings are sprinkled over a horizontal pane of glass, they will show a slight tendency to arrange themselves in lines parallel to the magnetic meridian, particularly if the glass be jarred. One might infer that the lines of force due to the earth's magnetism are horizontal. This is not, however, the case; the direction in which the lines are inclined is from north to south, according to the compass, but the lines make any angle with the horizon (§ 128); 70° or 80° for instance in the United States. We have already made use of the surveying-compass to find the magnetic meridian (¶ 183). The compass affords, however, little or no idea of the angle which the lines of force make with the horizon, because a compass-needle is suspended so as to move approximately in a horizontal plane.[1] To find the magnetic dip (§ 128), we may make use of an instrument known as the "dipping-needle." A simple form of this instrument consists of a knitting-needle *ad* (Fig. 210), with an axis *bc* soldered to it a right-angles and resting on two glass surfaces *b* and *c*, attached by sealing-wax to wooden supports (*be* and *cf*), and made horizontal by means of a spirit level.

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[1] If the throws exceed 30° the student should plot the *chords* of the angles in question (Table 3), instead of the angles themselves (see § 109).

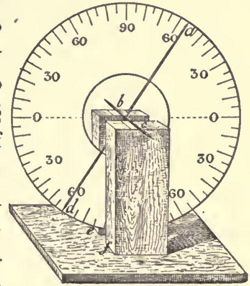

Fig. 210.

In practice the needle must be balanced by bending the axis *bc*, or by adding bits of sealing-wax or solder to it, so that it will stay, *when unmagnetized*, in any position, as *ad*. Then the needle is magnetized by stroking the end *a* ten times from the centre outward with the north pole of a powerful magnet, and by stroking the end *d* similarly with the south pole of the magnet. The needle will no longer balance in any position; but the north pole will, in north latitudes, dip downward as in Fig. 210. To measure the angle of the dip, a cardboard protractor, cut out at the centre so as not to interfere with the axis of the needle *bc*, is attached vertically to one of the wooden supports (*be*), and turned round so as to be north and south according to the compass. The axis *bc* is made to point horizontally east and west, and to coincide as nearly as possible with the axis of the graduated circle. The mean reading of the two ends (*a* and *d*) of the needle should then give correctly the angle of the dip. Errors of parallax must of course be guarded against (§ 25). Various other sources of error may be eliminated by a series of experiments. In some of these the axis *bc* should be turned end for end, in some the whole instrument should be turned end for end, and in some the magnetism of the needle should be reversed by stroking the end *d* upon the north pole, and the end *a* upon the south pole of a magnet. By averaging the various results, the angle of the magnetic dip may be determined within a few degrees.

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[1] The needles of surveying compasses intended for use in widely different latitudes are frequently provided with a small sliding weight by which variations in the magnetic dip and intensity may be counterpoised.

¶ 191. **The Earth Inductor.** — If a hollow square of wire *CDEF* is laid upon the floor with the side *CD* magnetically east and west, and rotated about *CD* as an axis into the position *ABCD*, it is evident that the wire *EF* must cut all the lines of force due to the earth's magnetism which pass through the areas *ABCD* and *CDEF*. The line *CD* will cut no lines of force, because it is stationary; and the wires *CE* and *DF* will cut none, because their motion is in a plane parallel to the lines in question. All the lines cut will therefore be included in the area $ABEF$.

If the square is now held against the west wall of the room, in the position $C'D'E'F'$, and rotated as before about an axis ($C'D'$) perpendicular to the lines of force, into the position $A'B'C'D'$, the number of lines cut will be as before included in the area $A'B'E'F'$; and similarly if the square is rotated about an axis $C''D''$, in the north wall of the room

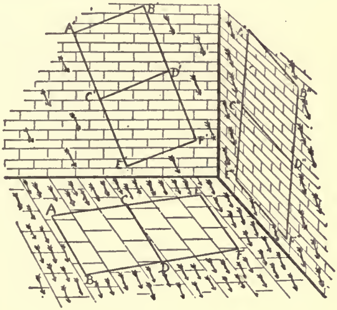

Fig. 211.

perpendicular to the lines of force, the lines cut will all be included in the area $A''B''E''F''$. Now the areas ($ABEF$, $A'B'E'F'$, $A''B''E''F''$) are all equal, — each being twice the area included by the square. If, therefore, we connect the terminals of the square with a galvanometer, and observe the throws of the needle which take place when the square is suddenly turned over, we shall have a means of comparing the relative numbers of the lines of force which pass through the square in its three different positions.

From these data we may infer the direction of the lines of magnetic force. If, for instance, the throw of the needle is much greater when the square is turned over on the north wall of the room than on the west wall, we may infer that more lines of force pass through the square in the former position; and that, accordingly, these lines are more northerly than westerly. If, again, the throw is much greater when the square is turned over on the floor than on either wall, we may infer that the lines of force are more nearly vertical than horizontal. We will suppose, for simplicity, that the walls of the room face exactly north and west by the compass, so that no lines of force pass through the loop when held against the west wall of the room.

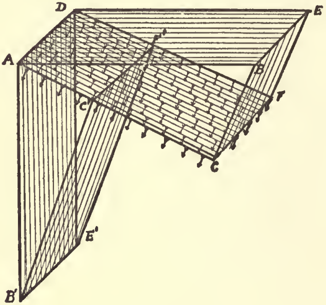

Fig. 212.

Let *ABED* and *AB′E′D* (Fig. 212) represent respectively the square in its horizontal and in its vertical position, AD being magnetically east and west; let the plane $ADF'FCC'A$ be drawn perpendicular to the lines of force, and the planes $BEFC$ and $B'E'F'C'$ parallel to the lines. Then the areas $ADFC$ and $ADF'C'$ include respectively the lines which pass through the square in its two positions. Since the lines are equally spaced, their numbers are as the areas which include them. These areas are to each other as $AC : AC'$, or since by construction $BC = AC'$, they are to each other as $AC : BC$. This ratio $(AC : BC)$ is by definition the tangent of the angle ABC, which measures the magnetic dip.

Now if a' is the angle through which the needle is thrown when a loop of wire is turned over on the floor, and if a'' is the same for the north wall of a room, the impulses given to the needle are to each other as the chord of a' is to the chord of a'' (see § 109), or approximately as a' is to a''. It follows that the angle of the dip a is given by the formula —

$$\tan a = \frac{\text{chord } a'}{\text{chord } a''} = \frac{a'}{a''} \text{ nearly.}$$

The same proportion will be found to hold for a round loop of wire. In practice we employ a coil of wire, containing, let us say 100 turns, since the effect upon the galvanometer increases with the number of turns.

The student should note that a sliding motion given to such a coil either along the floor or along the wall causes no deflection of the galvanometer. This is because the lines of force are cut by the two halves of the coil in opposite ways. It will be found to make no difference whether the coil is rotated about an axis passing through its centre, or on one side of it. We need to consider only the angle through which rotation has taken place. A coil capable of being thus rotated 180° about a horizontal and about a vertical axis constitutes what is called an "earth inductor," because of the currents of electricity which by the action of the earth's magnetism, may be "induced" in it.

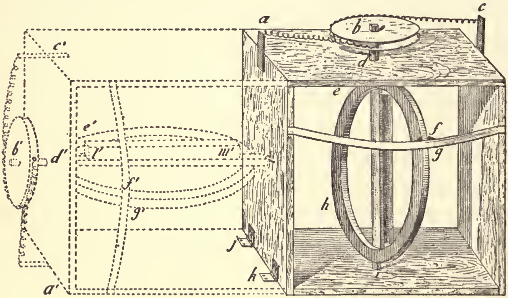

Fig. 213.

¶ 192. **Determination of the Magnetic Dip by means of an Earth Inductor.** — A convenient form of earth inductor is represented in Fig. 213.[1] It consists of a coil of wire *h*, mounted on a wooden axle *di*, with a head *b*, through which the coil may be set in rotation by the spring *cbd*. An auxiliary spring *ad* may also be employed to hasten the rotation through the first right-angle, and to slacken it in the second right-angle, so that the coil may be arrested by the catch *f*, when it has rotated through exactly 180°. By winding the spring *abd* round the head of the axle in the other direction, the coil may be made to return to its original position. The apparatus is permanently attached to the floor by means of two hinges *j* and *k*, the axes of which are east and west. If the coil is properly counterpoised, it will operate also when the whole instrument is tipped on its side, as represented by the dotted lines in Fig. 213.

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[1] The instrument may be greatly simplified if it is intended only to be turned by hand. This generally requires the co-operation of two students, one to turn the earth inductor properly, the other to observe the throws of the galvanometer.

Wedges are to be placed beneath the frame so that the axis of the coil may be exactly vertical in one position, and exactly horizontal in the other position. The catch *f* must be adjusted if necessary, so that the coil may be horizontal in the second position. If the hinges are properly placed the plane of the coil will be at right-angles to the magnetic meridian in both positions.

The axis of the coil is first to be made horizontal, and the terminals of the coil are to be connected (see ¶ 193, 11) with a galvanometer (Fig. 207, ¶ 188), placed at a considerable distance from the earth inductor so as to avoid jarring, and adjusted as in ¶ 189. The catch *f* is then to be lifted by pulling a string attached at *g*. The throw of the needle is to be noted. When the needle has come to rest (see ¶ 189, footnote) the coil is made to return suddenly to its original position by the same mechanism. The throw of the needle is again observed, and the mean throw (a') calculated.

The experiment is to be repeated with the axis of the coil vertical. The mean throw (a'') is to be found. The angle of the dip (a) is then to be calculated by the formula (see ¶ 191),

$$\tan a = \frac{a'}{a''}, \text{ nearly.}$$

ELECTRICAL MEASUREMENTS.

CURRENT STRENGTH.

¶ 193. **General Precautions in the Measurement of Electric Currents.** — Nearly all measurements of electric currents involve the use of galvanometers depending upon the deflection of a magnetic needle. The same precautions must accordingly be observed in electrical as in magnetic measurements.

(1) DELICACY OF SUSPENSIONS. A needle weighing less than 10 grams may be safely suspended by a single fibre of the best cocoon silk. When several fibres are employed they should be fastened together with wax, but *not twisted together.* If great delicacy is desired, the finest possible thread should be employed.

When a needle is hung on a pivot, as in an ordinary compass, great care must be taken to preserve the sharpness of the steel point upon which it turns. A lever should be arranged so as to lift the needle from the pivot when the instrument is not in use ; and when in use, care should be taken not to jar the compass. A slight jarring may be used as a last resort to relieve the friction between the needle and its pivot when the latter has been already dulled. It is preferable, when possible, to observe the turning-points of the needle while oscillating in a small arc, and from these to infer its position of equilibrium (see ¶ 20).

(2) PRESERVATION OF MAGNETISM. The needle of a galvanometer should be carefully protected from strong magnetic forces, whether due to permanent magnets or to electric currents, since such forces are apt to affect the magnetism of the needle. This precaution is especially important in the case of "astatic" needles (¶ 188), since the slightest change in either of the two parts of which such needles are composed may completely destroy the balance between them, and thus seriously injure the delicacy of the combination.

Strong currents should never be sent through delicate galvanometers. The terminals of such galvanometers (*a* and *b*, Fig. 214) should be joined together with a wire or "shunt" (*c*), forming a cross-connection between the wires (*d* and *e*) which convey the current to and from the galvanometer. An electric current of unknown strength should be first tested by the galvanometer *with the shunt*. If the galvanometer shows little or no deflection, the shunt may be safely removed.

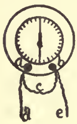

FIG. 214.

(3) MAGNETIC SURROUNDINGS. All iron, steel, or other magnetic substances should be removed, if possible, from the neighborhood in which magnetic measurements are to be performed. The positions of magnetic bodies which cannot be moved should be accurately noted. Especial care must be taken to guard against *changes* in the position of magnetic

bodies in a course of experiments.[1] The position of a galvanometer should be accurately located, since considerable variations, both in the direction and in the strength of the earth's magnetism, often occur in different parts of the same building, unless special care has been taken to avoid the use of iron in its construction. When there is no simpler way of describing the place of an instrument, its distances may be found from the floor and from two walls of the room.

(4) Rate of Oscillation. Any change in the strength of the magnetic forces acting upon a needle, in the magnetism of the needle itself, or in the freedom of its suspension will be found to affect its rate of oscillation. It is well, therefore, to determine this rate before and after every experiment in which such changes are likely to occur. This precaution is particularly important in the case of astatic needles and in the method of vibrations (Exp. 82).

(5) Excentricity. When a compass-needle is suspended at a point not exactly in the centre of the graduated circle by which its position is determined, errors due to "excentricity" may be introduced. Such errors are avoided by *reading both ends of the needle.*

(6) Zero-Reading. A galvanometer is always to be adjusted (except in the method of vibrations, Exp. 82) with the plane of its coil vertical, and parallel to the needle in its zero position, — that is, the position which the needle takes when no current is flowing

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[1] Students should be cautioned against carrying small objects made of iron or steel about their person.

through the coil. In the case of a galvanometer provided with an ordinary compass-needle, the plane of the coil is accordingly to be made parallel to the magnetic meridian. In this position the reading of the needle should be zero. It is well to make sure (§ 32) that the zero-reading is not disturbed in the course of an experiment, either by dislocation of the galvanometer or by changes in the position of magnetic bodies in the vicinity (see 3).

(7) MUTUAL INDUCTION. To prevent the coils of one instrument from affecting the needle of another instrument, these instruments should be separated as widely as may be practicable. In certain delicate experiments the effects of magnetism produced in one building are measured by electrical wires carried to an entirely separate building. Coils of wire are in general made horizontal if possible; magnets vertical; since in these positions minimum magnetic effects are usually produced on galvanometers in their vicinity.

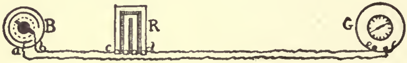

FIG. 215.

(8) CONNECTING WIRES. The wires conveying an electric current to and from an instrument should be parallel and close together, so that the equal and opposite currents in these wires may neutralize each other as far as magnetic effects are concerned. A typical case is represented in Fig. 215, where by the parallel wires *bc*, *de*, and *af*, a battery *B* is connected

through a rheostat R with a galvanometer G (see Exp. 92). It will be found convenient in practice to twist the wires together. In rheostats the wires are wound double (see Fig. 240, Exp. 86) to avoid magnetic effects.

(9) Reversal of Currents (§ 44). Every instrument capable of being affected by magnetic influences from outside should be provided with means of reversing the current through it, without changing its direction in other parts of the circuit. Any such instrument is called a "commutator." A convenient form of "commutator" is represented in Fig. 216.[1]

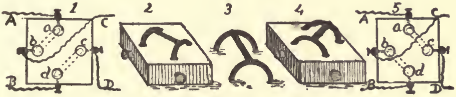

Fig. 216.

(10.) Waste of Power. The commutator may be made also to serve as a "key," — that is, to cut off the current from the battery. This is done by simply removing the rods (Fig. 216, 3) from the mercury cups. In the absence of a commutator or key, one of the battery wires should be disconnected when the battery is not in use, not only to prevent unnecessary waste of power, but also to avoid serious errors which may result either from the deterioration of the battery or from heating the wires.

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[1] This commutator consists of a square block of mahogany or ebonite, with four holes *abcd* (Fig. 216) bored half-way through it. The screws of four binding-posts are driven horizontally into these holes, which are then filled with mercury. Two copper rods (Fig. 216, 3), bound together by a handle of mahogany or ebonite, are bent so as to reach respectively either from a to b and from c to d, or from a to c and from b to d (see Figs. 216, 2 and 4). The wires (A and B) from the positive and negative poles of a battery are connected with two *opposite* mercury cups, as a and d; the wires C and D, leading to the instrument in which the current is to be reversed, are connected with the other pair of opposite cups (as b and c). It will be seen that in one position of the commutator (Fig. 216, 1 and 2), the wire A is connected with C, while B is connected with D; in the other position (Fig. 216, 4 and 5) A is connected with D, while B is connected with C.

When a battery is not required for several days it is well to empty out the fluids which it contains, each into a separate vessel, in which it may be preserved for future use, if not already exhausted. The zincs and coppers or carbons should be placed in pure water; the porous cups left to soak in a solution of dilute sulphuric acid so as to be ready for immediate use; the clamps, being disconnected from the poles of the battery, should be carefully cleaned and dried.[1]

(11) ELECTRICAL CONNECTIONS. All electrical connections depending upon metallic contact should be carefully examined. The metallic surfaces should be scraped bright and bound together with considerable pressure. A good electrical connection between two copper wires may generally be made by twisting them together. A soldered joint is to be preferred if the connection must remain good for an indefinite length of time. A liberal supply of binding-posts, screw-cups, and couplings, will be found of value in electrical measurements.

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[1] These remarks apply particularly to cells of the Daniell or Bunsen type (Figs. 234 and 235, Exp. 84). With a Leclanché cell (Fig. 236), these precautions are unnecessary.

The best temporary connection is undoubtedly made by dipping copper into mercury (see 9). The surface of the copper should first be amalgamated by dipping it into nitrate of mercury and rubbing it with a cloth.

(12) INSULATION. Care must be taken that electrical connections are not made when they are not wanted. The student should carefully examine the insulating material with which his wires are wound, particularly when the wires are to be twisted together. He should make sure that there is no current between any two of the binding-posts of a commutator or rheostat which can be detected by a galvanometer when the metallic connections are broken. The outside of battery cells should be dry for if they are not, electrical leakage is apt to take place. There is in fact more or less leakage in all experiments; but if the apparatus be perfectly dry this will probably not be enough to affect the accuracy of any of the measurements which follow.

EXPERIMENT LXXVIII.

CONSTANTS OF GALVANOMETERS.

¶ 194. **Construction of a Single-Ring Tangent Galvanometer.** — A form of galvanometer frequently employed, because of its simplicity of construction, is represented in Fig. 217. A horizontal cross section is given also in Fig. 218. The instrument consists of a compass (*a*, Fig. 217, and *dgif*, Fig. 218) mounted on a wooden support in the middle of a coil of insulated wire. The compass-needle (*eh*) is made very short[1] so that the *whole* of it may be virtually at the centre of the coil. To assist in reading the deflections of the needle, two long light pointers (*f* and *g*) are attached to it at right-angles. The wire is wound on a grooved brass ring in a single layer. The ends of the wire are carried to binding-posts (*c*, Fig. 217) at the base of the instrument as close together as possible. Levelling screws (*b* and *e*, Fig. 217) are usually added. In the construction of the instrument

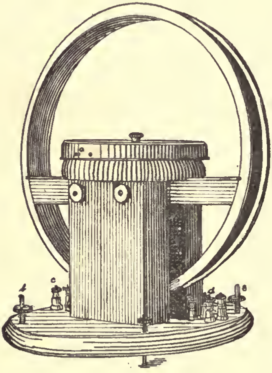

Fig. 217.

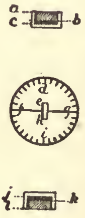

Fig. 218.

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[1] The length of the needle should not exceed $\frac{1}{12}$ the diameter of the coil. Kohlrausch, Physical Measurement, Art 63.

neither iron nor steel must be used (¶ 214, 3) except in the magnet itself, and in the steel pivot upon which it turns. The compass should have a lever to lift the needle from the pivot when the instrument is not in use (¶ 214, 1).[1]

¶ 195. **Law of Tangents.** — When an electrical current of sufficient strength is sent through the coils of a galvanometer, lines of magnetic force due to the current may be recognized by the the aid of iron-filings scattered upon a horizontal piece of glass. We will suppose that the plane of the coil is parallel to the magnetic meridian (that is, vertical, and magnetically north and south ¶ 214, 6), and that the glass passes through the centre of the coil. Lines of force will then be formed in a direction which, if the current is sufficiently powerful, may differ imperceptibly from east and west near the centre of the coil.

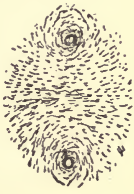

FIG. 219.

When a compass-needle is placed at the centre oi the coil, it takes a direction, as might be expected, parallel to the lines of force passing through that point. If we suppose the current to be ascending on the north side of the coil, and descending on the south side, the north pole of the needle will point nearly to the east. The electric current *tends* in fact to deflect the compass-needle due east and west, but the earth's magnetism combined with it always gives to the needle a more or less northerly direction.

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[1] Single-ring galvanometers in the Jefferson Physical Laboratory have been constructed with 10 turns of No. 16 insulated copper wire, wound on a brass ring 36 *cm.* in diameter. The supports are made of wood. The needle is $2\frac{1}{2}$ *cm.* long. The pointers are of aluminum, and each about 5 *cm.* long. The circle is divided into degrees and half-degrees. The coil is arranged in sections of 1, 2, 3, and 4 turns, with connections so that any number of turns can be employed from 1 to 10. By sending the current through these sections in different directions the sections may be tested against one another.

The actual direction of the compass-needle is determined (see ¶ 184) by two forces: one, H, due to the horizontal component of the earth's magnetism acting in a northerly direction; the other, F, due in this case, not (as in ¶ 184) to a magnet, but to the magnetic effect of the electrical current acting in an easterly or westerly direction. The angle (a) of deflection is given accordingly, as in ¶ 184, by the formula,

$$\frac{F}{H} = \tan a. \qquad (1)$$

The units of current now in use have been defined (§ 132) with reference to the magnetic field which a current produces in a coil of wire. If L is the length of the wire, R its mean radius, and c the current in absolute units, we have

$$F = \frac{cL}{R^2}. \qquad (2)$$

Or if C is the current in ampères (§ 19), we have—

$$F = \frac{1}{10}\,\frac{CL}{R^2}. \qquad (3)$$

Substituting this value in (1) we have—

$$\frac{CL}{10\,R^2H} = \tan a. \qquad (4)$$

Let us suppose that two currents C and C' produce the deflections a and a' respectively; then

$$\frac{C\,L}{10\,R^2\,H} = tan\ a; \qquad (5)$$

and

$$\frac{C'\,L}{10\,R^2\,H} = tan\ a'. \qquad (6)$$

Dividing (5) by (6) we find —

$$C : C' :: tan\ a : tan\ a'; \qquad (7)$$

that is, in a given galvanometer two currents are proportional to the tangents of the angles of deflection which they respectively produce. This is known as the *Law of Tangents.*

¶ 196. **Calibration of a Tangent Galvanometer.** — The single-ring galvanometer described in ¶ 194

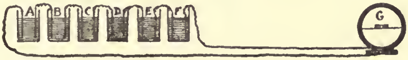

Fig. 220.

may approximate more or less closely to the conditions required of a perfect tangent galvanometer. To test the accuracy with which the "Law of Tangents" (¶ 195) is fulfilled, a battery of six small Daniell cells may be employed. The cells should be as nearly as possible of the same size and composition.

The plane of the galvanometer coil is to be made parallel to the magnet meridian (¶ 193, 6) so that the compass-needle points to 0° at both ends; then the two terminals are to be connected, with the poles of the battery arranged in series, as in Fig. 220, and in

Fig. 221, 1, so that the cells may all act together. The connecting wires should be well insulated (¶ 193, 12) and twisted together (¶ 193, 8). The deflection of the galvanometer is to be found by reading both ends of the needle (¶ 193, 5).

The connections of the poles of the first cell (*A*) are now to be interchanged (Fig. 221, 2) so that it acts against the other five. The deflection is to be found as before. Then the original connections of *A* are to be restored, but those of the second cell (*B*) reversed (as in 3), and the deflection again noted; and so in turn each cell is to be opposed to the rest (as in 4, 5, 6, and 7). Then *A* and *B* are both to be reversed (as in 8), then *C* and *D* (as in 9), then *E* and *F* (as in 10). The student may be interested to test the equality of the cells by opposing *A*, *B*, and *C* against *D*, *E*, and *F* (as in 11, or as in 12). In repeating the measurements, the connections of the galvanometer should be interchanged (¶ 193, 9), and the measurements should be repeated in the inverse order, to eliminate variations in the strength of the

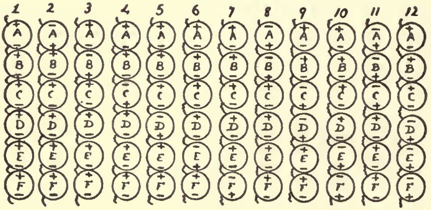

Fig. 221.

cells. The results are to be reduced as in ¶ 197, below.

¶ 197. **Reduction of Results of Calibrating a Tangent Galvanometer.** — In (1) we have six cells in series; in (2), (3), (4), (5), (6), and (7), we have in each case one cell opposed to five others or the equivalent of four cells. The *average* deflection gives, therefore, the effect of four cells of the same *average* strength as the six cells in (1). In (8), (9), and (10), we have in each case two cells opposed to four others, or the equivalent of two cells in all; the average deflection corresponds accordingly to two cells of the average strength.

In 11 and 12 there should be little or no deflection. Since the galvanometer is sensitive to the direction as well as to the magnitude of the current, the deflections in 11 and 12 should be equal and opposite.

The results are arranged in tabular form below:

1. No. of cells acting.	2. Average deflection.	3. Tangent of deflection.	4. Ratio of 3 to 1.
6	56°.5	1.511	.252
4	45°.3	1.011	.253
2	27°.1	.512	.256

We notice that the path of the electrical current is the same in all the arrangements, except that in some cases it passes through a given cell in one direction, in other cases in the opposite direction. It is stated that the electrical resistance of a cell is the same, regardless of the direction of the current.[1]

---

[1] Work is required to drive a current backward through a cell, whereas if a current passes through it in the ordinary direction, the cell is a source of power (see § 137). In calculating the *electrical re-*

The total electrical resistance is accordingly the same in each of the twelve arrangements shown in Fig. 221. It is also stated that the electro-motive force of a battery is proportional to the number of cells acting, hence by Ohm's law (§ 138) the ratio of the numbers in the third column to those in the second column should be nearly constant. If it is not, the galvanometer should be discarded for accurate purposes. The experiment should be repeated with a galvanometer in which the Law of Tangents is at least approximately fulfilled.

¶ 198. **Determination of the Constant of a Single-Ring Galvanometer.** — It is evident from formula 4, ¶ 195, that the deflection of a galvanometer depends not only upon the electrical current, but also upon the length and radius of the coil of wire through which it flows. In order to measure currents with a galvanometer, it is therefore necessary to determine

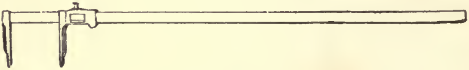

Fig. 222.

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sistance of a cell we do not consider the gain or loss of power due to chemical agency, but only the loss of power due to conversion into heat. The statement that the resistance of a cell is the same without regard to the direction of the current does not mean, therefore, that it is as easy to drive a current backward through it as to drive it forward, but that the cell would be *equally heated* in both cases. The truth of this statement has recently been called into question, but the method of calibration described above has been found practically to yield accurate results.

accurately the dimensions of the coil of wire. To find the diameter of a coil, we measure with a long vernier gauge (Fig. 222) the distance between the flanges of a bobbin (*al*, Fig. 223) upon which the coil is wound. Then we find the thickness of two blocks *ab* and *kl* which fill the space between the wires and the edges of the flanges. Subtracting *ab* and *kl* from *al* we have the outside diameter (*bk*) of the coil. We now measure the width of the bobbin and the width of the flanges. Subtracting the latter from the former, we have the width of the coil of wire. The whole number of turns of wire is now to be counted. Usually the groove is broad enough for one more turn of wire than that actually wound upon it, since this amount of space is necessary for turning the wire. The width of the groove is to be divided by the number of turns which would fill it, to find the average diameter (*bc*, or *jk*) of the wire. Subtracting this from the outside diameter (*bk*) we have the mean diameter (*bj*, or *ce*) of the coil. Dividing by 2 we have the mean radius of the coil.

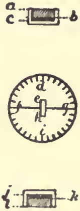

FIG. 223.

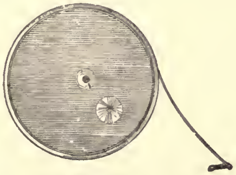
FIG. 224

Instead of measuring the diameter of the coil, we may find its circumference by passing a thin steel tape graduated in *mm.* around the outside of the coil. If *c* is the circumference, the outside diameter is

$c \div \pi$. From this the mean diameter and radius may be calculated as before. The results are to be still further reduced as in ¶ 199.

¶ 199. **Calculation of the Constant and Reduction Factor of a Tangent Galvanometer.** — The constant (K) of a coil of wire is equal to the ratio of its length to the square of its radius (§ 133). That is, in the notation of ¶ 195,

$$K = \frac{L}{R^2}. \qquad (1)$$

Substituting this value in formula 4, ¶ 195, we have

$$\frac{CK}{10\,H} = tan\ a, \qquad (2)$$

or solving for C,

$$C = 10\ \frac{H}{K}\ tan\ a. \qquad (3)$$

The constant, K, of a given galvanometer is therefore an important factor in the calculation of a current from the deflection which it produces in that galvanometer.

If n is the number of turns in the coil,[1] we have

$$L = 2\,\pi\,n\,R, \qquad (4)$$

which substituted in (1) gives

$$K = \frac{2\,\pi\,n\,R}{R^2} = \frac{2\,\pi\,n}{R}. \qquad (5)$$

---

[1] The student must remember that when a coil is made in two parts, so that half the current flows through each, the effect is the same as if the whole current flowed through one half. The total number of turns must therefore be halved in order to find the effective number n.

By this formula the constant of the tangent galvanometer is to be calculated. Thus for 6 turns of radius 18 *cm.* we have a constant $2 \times 3\frac{1}{7} \times 5 \div 18$, or 1.75, nearly. With such a galvanometer, assuming that the horizontal intensity of the earth's magnetism is 0.175, nearly, we should have from (3) —

$$C = 10 \times \frac{.175}{1.75} \, tan \; a = tan \; a \text{ (nearly)};$$

that is, the current in ampères would be numerically equal to the tangent of the angle of deflection produced.

In most galvanometers this is not the case. To find the current, we have to multiply the tangent of the angle of deflection by some factor, which may be greater or less than unity. This is called the *reduction factor* of the galvanometer.[1]

Denoting it by I, we have from (3) —

$$I = 10 \, \frac{H}{K}. \qquad (6)$$

It is important to find the reduction factor of a galvanometer which is to be used often, since it greatly shortens the reduction of results.

Substituting from (6) in (3) we have simply —

$$C = I \, tan \; a. \qquad (7)$$

It may be observed that if $a = 45°$, so that *tan a*

---

[1] Some writers call the reduction factor "the constant" of a galvanometer. Since the reduction factor depends upon the earth's magnetism (see 6), it is evidently not constant. The effect of changes in the earth's magnetism in a short course of experiments may, however, generally be disregarded.

$= 1$, we have $C = I$. The reduction factor of a galvanometer is therefore numerically equal to the current which deflects it 45°; that is, the current which produces a field of force at the centre of the coil equal to the horizontal component of the earth's magnetism.

EXPERIMENT LXXIX.

COMPARISON OF GALVANOMETERS.

¶ 200. **Construction of a Double-Ring Tangent Galvanometer.** — A "double-ring" tangent galvanometer is represented in Fig. 225, also in horizontal section in Fig. 226. It consists of two parallel coils of wire wound on brass or wooden rings *a* and *b*, with a surveying-

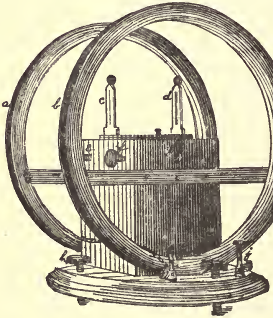

Fig. 225.

compass *cd* between them (see also Fig. 199, ¶ 183). In the case of a single-ring galvanometer, it has been stated that the length of the needle should not exceed $\frac{1}{12}$ the diameter of the coils. In the double-ring galvanometer, it may be $\frac{1}{4}$ of this diameter without introducing any serious error into the results (Kohlrausch, Art. 93). For measuring battery currents, each coil should contain about six turns of No. 12 insulated copper wire. It is recommended that the average diameter of the coils should be 32 *cm.* and the mean distance between them 16 *cm.*[1] The needle of the surveying-compass should be not more than 8 *cm.* long. When a current is made to divide in such an instrument into two parts, so that half flows through each coil, it is found that the tangent of the angle of deflection is approximately equal to the magnitude of the current in ampères.

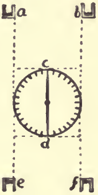

FIG. 226.

¶ 201. **Determination of the Reduction Factor of a Galvanometer by the Method of Comparison.** — The single-ring galvanometer (Fig. 217) is to be adjusted with its coil north and south (¶ 193, 6), as near as possible to the place (¶ 193, 3) where the horizontal intensity of the earth's magnetism was determined (¶ 183). The double ring galvanometer (Fig. 225) is to be similarly adjusted in some position convenient for future measurements. This position should be accurately noted. The two instruments (A and C, Fig. 227) are then to be connected in series with a constant battery (B) capable of yielding a current of one or two ampères. The deflection of each galvanometer is to be found by reading both ends of each needle (¶ 193, 5). The connections of C are then reversed (see ¶ 193, 9), and both deflections again noted. The connections of A are next reversed and new readings taken. Finally the connections of C are again reversed, so as to be the same as at the start,—the needles being read as before.

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[1] These dimensions have been calculated for places where the horizontal component of the earth's magnetism is .169 or .17 nearly. In places where this horizontal component is nearly .18 the dimensions should be 30 and 15 *cm.* respectively.

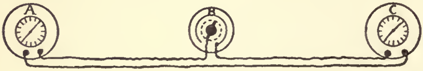

Fig. 227.

The observations of the two galvanometers should be made at the same time, as nearly as possible. Let a be the average angle through which A is deflected; a' that through which C is deflected; then if the reduction factors (¶ 199) of A and C are I and I' respectively, the current C which traverses both galvanometers must be (see ¶ 199, formula 7) —

$$C = I \tan a = I' \tan a';$$

hence the reduction factor (I') of C may be found by the equation —

$$I' = I \frac{\tan a}{\tan a'}.$$

We notice that the reduction factors of two galvanometers are to each other *inversely* as the tangents of the angles of deflection produced by a given current.

The student should be cautioned *not* to connect the two galvanometers in multiple arc (§ 140); for in this case the current divides into two parts, which may or may not be equal. Not knowing the ratio between the two parts, we can draw no conclusion as to the relative sensitiveness of the two galvanometers.

When the instruments are connected as above *in series*, the same current (if there is no leakage) must traverse the coils of both.

EXPERIMENT LXXX.

THE DYNAMOMETER.

¶ 202. **Construction of a Dynamometer.** — A form of dynamometer useful for measuring battery currents is represented in Fig. 228. It consists of a wooden bobbin, *fgpn*, with two grooves, in each of which are wound 50 turns of No. 16 insulated copper wire. Small holes are bored through the bobbin at *f*, *g*, *n*, and *p*, so that it is possible to measure directly the inner and outer diameters of the coil. The average diameter is about 25 *cm*. A small hollow wooden cube (*ijkl*), measuring 5 *cm.* each way, is now wound with 80½ turns of No. 24 copper wire, the ends of which

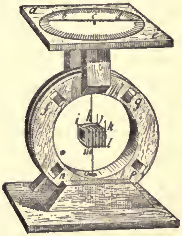

Fig. 228.

are connected by No. 31 spring brass wires (*ch* and *mo*) to a fixed point beneath, *o*, and to the centre (*c*) of a knitting needle (*bd*), as in the torsion balance (see Fig. 176, ¶ 165). The length of the wire should be taken so that the coefficient of torsion of the wire *ch* may be some round number, let us say 10 dyne-centimetres, per degree (see ¶ 165). Thus if 100 *cm.* of the wire has been found (Exp. 64) to have a coefficient of torsion of 2 dyne-centimetres per degree, we may make *ch* just 20 *cm.* long, so that it may exert a couple of $\frac{100}{20} \times 2 = 10$ units per degree.

It will be observed that the constant of the large coil, having in all 100 turns, and a mean radius of 12·5 *cm.*, is (see ¶ 133) —

$$K = \frac{2 \times 3.1416 \times 100}{12.5} = 50, \text{ nearly,} \qquad (1)$$

while the magnetic area of the smaller coil is (see § 134) —

$$A = 80\tfrac{1}{2} \times 5 \times 5 = 2000, \text{ nearly.} \qquad (2)$$

The constant of the dynamometer is accordingly (§ 135) —

$$D = 50 \times 2{,}000 = 100{,}000 \text{ absolute units, nearly.} \quad (3)$$

In other words, a current of 1 *absolute unit* would create a couple of 100,000 units, tending to twist the wire. A current of 1 ampère (being $\frac{1}{10}$ of the absolute unit) will have $\frac{1}{10}$ the effect, not only in the cube (*ijkl*), but also in the large coil (*fgpn*). The couple produced, depending upon the product of these two effects (see §§ 133, 134), will be accordingly less than D (in formula 3), in the proportion of 100 to 1. It follows that 1 ampère will exert in this instrument a couple of about 1000 dyne-centimetres; and that it will require a twist of 100° in the wire *ch* to balance it if, as has been supposed, 1° corresponds to 10 dyne-centimetres. Since the couple produced is proportional to the square of the current (§ 135), the current must be proportional to the square root of the angle of torsion which is required to balance this couple.

The proportions of the dynamometer have been chosen above so that the square root of the number of degrees indicated by the needle *bd* may give at once (approximately at least) the current in tenths of an ampère.

¶ 203. **Determination of the Constants of a Dynamometer.** — Before making use of a dynamometer to measure electrical currents, it is necessary to find (1) the constant of the large coil (*fgpn*, Fig. 228), (2) the magnetic area (§ 134) of the small coil (*ijkl*), and (3) the coefficient of torsion of the wire.

(1) The diameter of the large coil may be determined as in ¶ 198; but as the coils of the dynamometer contain several layers of wire, it is more accurate to measure directly the outside and inside diameters. For this purpose holes are made at *f*, *g*, *n*, and *p*, in the side of the bobbin. The number of turns, if unknown, may be estimated by counting the layers and the number of turns in each. From the whole number of turns and from the mean diameter of the coil, the constant (K) is to be calculated as in ¶ 199.

(2) To find the mean diameter of the square coil, the outside diameters *jk* and *kl* are to be measured by a Vernier gauge. The diameter of the wire is to be found by measuring the width of the 80 or more turns between *i* and *j*, then dividing by the number of turns. Subtracting this diameter from the outside diameters *jk* and *kl*, we have the mean diameter of the coil. Unless a wire passes through the middle of the cube in the direction *co*, it is obvious that there must be a whole number of turns plus one half turn on the cube *ijkl*. To avoid making a mistake, the turns should be counted on both sides of the cube. The magnetic area, A, of the square coil is then calculated as in § 134.

(3) The instrument is now to be laid upon its side, and a light balance-arm is to be attached to the cube (see Fig. 176, ¶ 165). The wire *ch* will probably have to be supported near *h* to prevent it from sagging under the weight of the cube. The wire should, however, rest freely upon the support, so as not to affect the torsion. The coefficient of torsion of the wire *ch* is then to be found as in ¶ 165.

¶ 204. **Determination of Reduction Factors by means of a Dynamometer.** — The Dynamometer is now to be set upright with the plane of the large ring north and south, and adjusted by twisting the needle *bd* so that the planes of the large and small coils are at right-angles. A fixed mark should be placed on the wall of the room so as to be in line with two sights *jk* on the small coil, when the coil is at right-angles to the large coil. The reading of the needle is to be observed. The instrument is then to be connected (as in ¶ 201) in series with a single-ring tangent galvanometer, and with a battery of several Bunsen cells, capable of sending a current of about 1 ampère through the circuit. The needle *bd* is to be turned until the sights *j* and *k* on the small coil come in line with the same mark as before. The reading of the needle is to be again observed, and also that of the tangent galvanometer.

The current is now to be reversed in the large coil, but not in the small coil of the dynamometer; then reversed in the battery; then the original connections of the dynamometer are to be restored. In each case readings of the dynamometer and of the galvanometer are to be made.[1]

If t is the coefficient and a the angle of torsion of the wire, the couple is ta. If K is the constant of the large coil, A the magnetic area of the small coil, we have for the current c, by § 135 —

$$c = \sqrt{\frac{ta}{KA}}, \text{ in absolute units;}$$

or in ampères, $$C = 10\sqrt{\frac{ta}{KA}},$$

since an ampère is one tenth of an absolute unit.

From the current, C, and the mean deflection, d, which it produces in the tangent galvanometer, we may find the reduction factor of the latter by the formula —

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[1] The couple produced by a current may also be measured by turning the instrument on its side as in ¶ 203, 3, and directly counterpoising the current with weights placed in one pan of the balance.

$$I = \frac{C}{tan\ d}.$$

We may also find the horizontal component (H) of the earth's magnetism by the formula —

$$H = \frac{IK}{10},$$

derived from ¶ 199, 6, using the new value of I.

If the values of I and H found by means of the dynamometer differ from those previously determined (Exps. 74 and 78) by more than 5 or 10 %,[1] the student should repeat all the measurements upon which these values depend.

EXPERIMENT LXXXI.

ELECTRO-CHEMICAL METHOD.

¶ 205. **Determination of the Reduction Factor of a Galvanometer by the Electro-Chemical Method.** — The galvanometer is to be adjusted with the plane of its coil parallel to the magnetic needle (¶ 193, 5), and its exact position noted (¶ 193, 3). The terminals of the galvanometer (*h* and *i*) are to be connected with the poles of a Daniell cell, *a* and *b* (Fig. 229, 2), through a commutator *defg* (see ¶ 193, 9). The ordinary copper (or positive) pole is replaced by a spiral of copper wire (*b*, Fig. 229, 1 and 2) with a coupling *c*, provided for convenience in weighing. The spiral should have been cleaned with nitric acid before the experiment. The solution of sulphate of copper with which it is surrounded should be saturated and free from all impurities, especially acid, ammoniacal, and oxidizing or reducing agents. The deflection of the galvanometer should be about 45, — more rather than less. If it is less than 30° the porous cup should be changed, or another cell substituted. When the spiral has been freshly coated with copper by the action of the battery, it should be disconnected from the coupling (*c*), dipped in three changes of fresh water, then in alcohol, and dried in a temperature not exceeding 100°, to avoid oxidation of the copper. Its weight is then to be found within a milligram, if possible, by a series of double weighings (Exp. 8).

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[1] The use of a small square coil in a dynamometer is simply for convenience in the explanation of the instrument to students. For accurate measurements, a round coil is to be preferred. In any case there are certain corrections to be applied to the dynamometer on account of the size and shape of its coils (unless these be carefully proportioned) which if neglected may account for errors of 3 or 4 %.

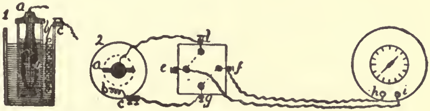

FIG. 229.

The spiral is now to be replaced in the cell, and connected with the galvanometer as before. The time when the connection is made must be accurately noted. The deflection of the galvanometer is to be recorded at intervals of one minute. Each end of the needle should be alternately observed (¶ 193, 5). At the end of $25\frac{1}{2}$ minutes the commutator *defg* is to be suddenly turned (see ¶ 193, 9) so that the current through the galvanometer may be reversed. Observations of the galvanometer needle are to be continued, at intervals of one minute, for another 25 minutes. There will thus be 50 observations in all. At the end of 50 minutes and 50 seconds, exactly, the current is to be suddenly cut off. The copper spiral is to be cleansed in three changes of water, with care not to dislodge any of the fresh deposit, then dipped in alcohol, dried, and reweighed accurately as before. The results are to be reduced as in ¶ 230.

¶ 206. **Theory of the Electro-Chemical Method.** — It has been found that a current of 1 ampère deposits 1 gram of copper in the course of 50 minutes and about 50 seconds (the total duration of the experiment). The strength of the solution has little or no effect upon the result, always provided that *enough* copper is present in it (§§ 142, 143). The amount of copper deposited varies only with the strength and duration of the current.

If C is the strength of the current in ampères, t the time in seconds, and w the weight of copper deposited, we have accordingly —

$$w = \frac{Ct}{3050}, \text{ nearly,} \qquad (1)$$

and $$C = \frac{3050\,w}{t}, \text{ nearly.} \qquad (2)$$

If, as in the experiment, $t = 50$ minutes and 50 seconds, that is, 3050 seconds, we find simply —

$$C = w. \qquad (3)$$

That is, the average value of a current in ampères is numerically equal to the weight in grams of copper deposited by it in 3050 seconds.

Now from ¶ 199, 7, we have, at any point of time,

$$C = I \tan a, \qquad (4)$$

where a is the angle of deflection produced by the current in a tangent galvanometer, and I is the reduction factor of the galvanometer. Hence, averaging the different results from the 50 observations of the needle, we find, comparing (3) and (4) —

$$w = \text{average of } I \tan a. \qquad (4)$$

In practice, if the angles do not differ by more than 10 %, the same result (nearly) may be obtained much more easily by averaging the angles themselves, then finding the tangent of this average. That is, if A is the average angle of deflection —

$$w = I \tan A, \text{ nearly.} \qquad (6)$$

The reduction factor may now be calculated by the formula —

$$I = \frac{w}{\tan A}. \qquad (7)$$

Having found the constant, K, of the galvanometer (¶ 199, 1), we may calculate the horizontal component (H) of the earth's magnetism, as in ¶ 204, by the formula (derived from ¶ 199, 6) —

$$H = \frac{IK}{10}. \qquad (8)$$

If the value of H obtained by the electro-chemical method does not agree with previous determinations (Exps. 74, and 80), the last experiment (Exp. 81) should be repeated until at least 3 results, obtained either by the same or by different methods, agree within let us say 5 %. All previous measurements leading to a different result should now be repeated.[1]

EXPERIMENT LXXXII.

METHOD OF VIBRATIONS.

¶ 207. **Construction of a Vibration Galvanometer.** — A form of galvanometer easily constructed is represented in Fig. 230. It consists of a coil *cfg* (made by winding 14 turns of No. 18 insulated copper wire upon a hoop of wood, brass, or pasteboard, 10 *cm.* in diameter) with a short magnetized needle *e*, attached to a bullet *d* and suspended at the centre of the coil by a fine waxed fibre (*cd*) of untwisted silk (see ¶ 186). The strength of the magnet and the weight of the bullet should be proportioned so that the needle may complete 10 vibrations in about 1 minute. A short test-tube may be employed to cut off currents of air (see Fig. 204, ¶ 186).

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[1] The student will do well to examine his *calculations* before repeating the measurements upon which they depend. A common error is a miscount or misconception of the number of turns of wire utilized in the coil of a galvanometer or dynamometer, particularly when the coils are connected in multiple arc. See footnote, ¶ 199.

The ends of the coil may be carried to binding-posts, *f* and *g*. Connections at *f* and *g* may also be made by simply twisting the wires together (¶ 193, 11).

When an ordinary battery current is sent through the coil, the magnetic field of force created by the current will greatly increase the rate of vibration of the needle. We have seen (¶ 186 and § 110) that a field of magnetic force is proportional to the *square* of the number of vibrations which it produces in a magnetic needle. In accordance with this law, the dimensions of the instrument have been chosen so that the square of the number of vibrations completed in 1 minute may represent approximately the strength of the current in thousandths of an ampère.

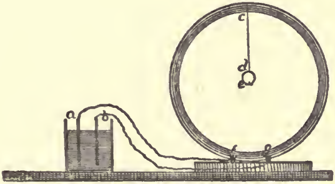

Fig. 230.

In calculating these proportions, it was assumed that the needle made exactly 10 vibrations per minute under the influence of the earth's magnetism, the strength of which was taken as 0.176 dynes per unit of magnetism (see Exps. 72, 73, 74, 80, and 81). No allowance was made for the effects of magnetism induced in the needle, which (unless the needle be of the best steel and highly magnetized) may account for errors of 5 or 10 per cent with currents of 1 or 2 ampères. To obtain accurate results with a vibration galvanometer, it would be necessary both to calibrate it (see ¶ 196) and to compare it (as in Exp. 79) with a galvanometer of known reduction factor. When, however, as in this experiment, the instrument is to be used for rough work and for relative indications only, such tests need hardly be applied.

The influence of the earth's magnetism upon the vibration galvanometer must be allowed for, as will be explained in ¶ 209.

¶ 208. **Determination of the Relative Strength of Battery Currents by means of a Vibration Galvanometer.** — A vibration galvanometer (¶ 207) is to be set up with the plane of its coil vertical, but (contrary to the usual custom, ¶ 193, 5) at right-angles with the magnetic meridian. The time required for 10 vibrations of the needle (which should be about 1 minute) is now to be accurately determined. The needle may be set in vibration by bringing a magnet near it, then suddenly taking the magnet away. The arc of vibration should not exceed 30 or 40 degrees (see Table 3, *g*).

The terminals of the galvanometer, *f* and *g*, are now to be connected respectively with the poles, *a* and *b*, of a battery constructed as will be described below. The student must notice carefully whether the needle points in the same direction as before, or whether the needle is reversed. In the latter case the connections of the galvanometer with the battery should be interchanged; that is, *f* should be connected with *b*, and *g* with *a*.

The number of vibrations made in 1 minute (or whatever time was required for 10 vibrations under the earth's magnetism) is now to be accurately determined. In no case should the arc of vibration exceed 30 or 40 degrees.

The battery to be employed in this experiment consists of a glass tumbler, half-filled with dilute sulphuric acid[1] (10 % by weight), a porous cup with an internal diameter not less than 5 *cm.*, containing a solution of sulphate of copper, and two strips, one of sheet zinc, the other of sheet copper, each 5 by 10 *cm.* Connecting wires should be soldered to both strips. The current from this battery is to be tested under the following conditions:

(1) When the zinc and copper strips are placed side by side in the sulphuric acid, but not touching each other.

(2) The same after the zinc has been amalgamated by rubbing it with mercury.

(3) (4) (5) The same after the current has been allowed to flow for five, ten, and fifteen minutes respectively.

(6) The same except that the bubbles gathered

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[1] To avoid accidents in mixing sulphuric acid with water, the acid should be poured in a fine stream into the water, so that the heat generated may be quickly dissipated.

on the copper strip have been removed by a camel's-hair brush, without exposing the copper to the air.

(7) The same, except that the copper has been exposed for a few minutes to the air.

(8) The same except that the copper has been amalgamated by being rubbed with nitrate of mercury.[1]

(9) The zinc and copper strips are now to be carefully weighed; the zinc is to be replaced in the sulphuric acid, but the copper is to be immersed in the solution of sulphate of copper contained in the porous cup, and the latter is to be placed in the tumbler containing the acid.[2]

(10) (11) (12) The same after the current has been allowed to run for five, ten, and fifteen minutes respectively. The zinc and copper strips are now to be reweighed. The results are to be reduced as will be explained in the next section.

¶ 209. **Reduction of Results obtained with the Vibration Galvanometer.** — It has been stated that the square of the number of vibrations completed in one minute by a vibration galvanometer constructed as in ¶ 207, gives approximately the current to which these vibrations are due in thousandths of an ampère. To find, accordingly, the current in ampères, we square the number of vibrations produced in the given length of time, and divide by 1000.

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[1] Copper may also be amalgamated by dipping it into nitric acid, then rubbing it with mercury by means of a cloth. Care must be taken not to let nitric acid come in contact with the hand.

[2] This combination constitutes a Daniell cell. See also Fig. 235, ¶ 211.

It must not, however, be forgotten that the earth's magnetism alone accounts for about 10 vibrations per minute. The earth's field is accordingly equivalent to that produced in the vibration galvanometer by $\frac{100}{1000}$ or 0.1 ampère. Care should have been taken in the experiment to have the earth's magnetism and the current acting always in the same direction. In this case all the results will be too great by 0.1 ampère. By subtracting this amount in each case, the effect of the earth's magnetism will be eliminated.

The strength of each current in ¶ 208, (1) to (12), should be calculated roughly in this way.

The student will notice that the visible action of the sulphuric acid on the zinc is arrested by amalgamating the zinc with mercury; that the action begins again when the zinc is connected with the copper strip, but that the bubbles of gas are then set free from the copper instead of from the zinc; that the amalgamation of the zinc does not impair the usefulness of the battery; that the current steadily decreases when both strips are in sulphuric acid, though it is temporarily increased by removing the bubbles from the copper, and by exposing the copper to the air; that amalgamation of the copper does not prevent the formation of bubbles upon it, nor improve in any way the action of the battery; that the formation of bubbles is arrested by placing the copper in the solution of sulphate of copper, and that in this case the battery furnishes a steady current; that the zinc plate loses in weight, but that the copper plate gains in weight by a nearly equal amount,[1] owing to fresh copper deposited upon it. We have already made use (in Exp. 81) of the quantity of copper thus deposited to measure an electrical current.

EXPERIMENT LXXXIII.

THE AMMETER, I.

¶ 210. **Testing an Ammeter.** — The name "ammeter" (an abbreviation of ampère-meter) is given to any instrument indicating directly the strength of electrical currents in ampères. Ammeters are manufactured in various forms. Most of them depend upon the attraction which an electrical current, circulating in a coil of wire (*b*, Fig. 231), exerts upon a permanent magnet or upon a core of soft iron. In some instruments this electro-magnetic attraction is balanced by a spring, in others by gravity; in others again it is balanced by the attraction of a permanent magnet (*c*). Such instruments depend for their accuracy upon the constancy of the magnet, and even if correctly graduated at the start, are subject to errors which may be indefinitely great. Recently instruments have been

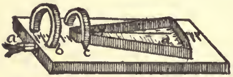

Fig. 231.

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[1] If we assume that there is no wasteful action of the battery, the quantities of zinc dissolved and of copper deposited should be to each other as the atomic weights of zinc and copper, 64.9 and 63.1 respectively.

manufactured in which currents are measured by the attraction between two coils of wire traversed by the same electrical current. Such instruments are properly called electro-dynamometers (see Exp. 84). If carefully graduated, they may serve as standards for the determination of electrical currents.

Ammeters are usually intended to measure currents of at least 10 ampères, and being generally sensitive only to about $\frac{1}{10}$ ampère, they cannot measure small currents very precisely. On the other hand, the tangent galvanometers described in ¶ 194 and ¶ 200 are intended to measure currents of a few ampères only. To compare an ammeter with such instruments, it must be connected with two or more of them in *multiple arc* (§ 140). A powerful battery of three or four Bunsen cells is then included in the circuit. A diagram of connections is given in Fig. 232, where *A* represents the ammeter, *BB* the battery, *C* and *D* two galvanometers. To avoid the influence of the connecting wires upon the instrument (¶ 193, 8), the arrangement would practically be made as in

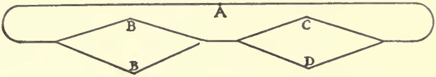

FIG. 232.

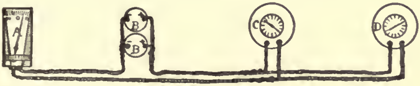

FIG. 233.

Fig. 233. The battery cells are represented in both diagrams (Figs. 232 and 233) as being connected in multiple arc (§ 140), since in this way they usually yield the greatest current through instruments of low resistance (§ 146).

If a, a', &c., are the deflections of the galvanometers; I, I', &c., their reduction factors, the currents through them are respectively $I \tan a$, $I' \tan a'$, &c. Hence the total current C is —

$$C = I \tan a + I' \tan a' + \&c.$$

The experiment should be repeated with batteries containing different numbers of cells, or the same number differently arranged, so as to produce currents of from 1 to 10 ampères.

The results should be tabulated in the ordinary manner, in three columns, containing respectively, (1) the current calculated from the galvanometer deflections; (2) the current indicated by the ammeter, and (3) the corresponding correction of the ammeter.

EXPERIMENT LXXXIV.

THE AMMETER, II.

¶ 211. **Determination of Battery Currents by means of an Ammeter.** — The electrical resistance (§ 136) of ammeters is usually so slight that it may be neglected. To measure the maximum current which a battery can produce, the screw-cups of the ammeter are to be connected by short thick copper wires with the pole-cups of the battery in question. The wires should be parallel or twisted together, as in the last experiment (see Fig. 233), and scraped bright at both ends (¶ 193, 11). The indication of the instrument is to be noted.

With any instrument of the class known as ammeters, the student is to determine the maximum current which can be derived from various well-known forms of voltaic battery, as, for instance, the Bunsen cell (Fig. 234), the Daniell cell (Fig. 235) and the Leclanché cell (Fig. 236). The observations may be continued in each case at intervals of five minutes for half an hour.[1] The material employed in each cell, and the dimensions of every part,[2] should be

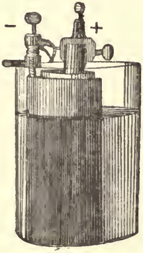

FIG. 234.

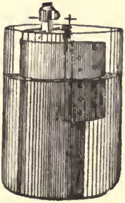

FIG. 235.

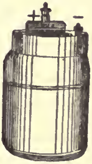

FIG. 236.

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[1] An old Leclanché cell may be employed for this experiment. It may serve subsequently for experiments with Wheatstone's Bridge, but for other purposes it will be rendered nearly useless.

[2] If a sufficient current cannot be obtained from a single cell of a given sort, two or more cells should be employed. The student should notice that with instruments like the ammeter having a very low resistance, it is more effective to arrange batteries in multiple arc than in series. See § 136, also Figs. 232 and 233, ¶ 210.

carefully noted. The corrections for various currents indicated by the ammeter have been found in the last experiment. The proper correction should be applied to each reading. The results are to be represented by a series of curves (Fig. 237) plotted on the same sheet of coordinate paper. A scale at the top of the paper indicates the time in minutes, and a scale at the left of the paper represents the current in ampères. Each curve should be marked with the name of the cell or battery to which it belongs.

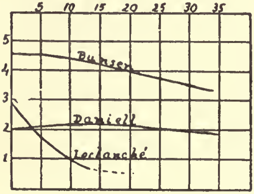

Fig. 237.

ELECTRICAL RESISTANCE.

EXPERIMENT LXXXV.

METHOD OF HEATING.

¶ 212. **Determination of Resistances by the Method of Heating.** — A short spiral (*a*, Fig. 238) of fine German silver wire, .01 *cm.* diameter (about No. 36) and 15 *cm.* long, is soldered to the two terminals *b* and *c* of two insulated copper wires, *d* and *e*, passing through a cork fitting the inner cup of a calorimeter (*B*, Fig. 239). The wires (*bd* and *ce*) should be so thick that their electrical resistance may be neglected in comparison with that of the spiral. The cork and wires are then inverted and placed in the calorimeter (*B*, Fig. 239) containing a sufficient quantity of distilled water to cover the spiral. The temperature of the water, which should be slightly below that of the room, is found by a series of observations (¶ 92, 10) made with a thermometer passing through the cork as in Fig. 239. The thermometer is provided with a stirrer (see ¶ 65, Fig. 50) so that a uniform temperature may be maintained.

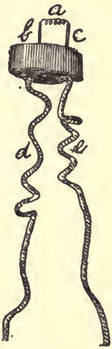

FIG. 238.

The instrument thus constructed (*B*, Fig. 239) is to be connected in series with a Bunsen cell (A) and with a tangent galvanometer (C) adjusted in the same place and manner as in Exp. 83.

The time when the connection is made must be accurately noted. The tangent galvanometer is to be observed at intervals of one minute. Between the observations, the water in the calorimeter is to be stirred by twisting the stem of the thermometer. When the temperature reaches that of the room, the direction of the electrical current is to be suddenly reversed by interchanging the battery connections (see ¶ 193, 9). The observations of the galvanometer are to be continued until the temperature of the water rises as high above that of the room as it was originally below it. Then the circuit is to be broken. The time when the current is interrupted must be accurately recorded. Several more observations of the temperature within the calorimeter are to be made at intervals of one minute, so that the resulting temperature may be accurately determined.

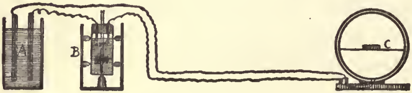

Fig. 239.

The weight of the calorimeter and of the water which it contains are finally to be found by weighing the calorimeter with and without the water.

¶ 213. **Calculation of Resistance by the Method of Heating.** — Let w be the weight of water, and W that of the calorimeter from which its thermal capacity

c is to be calculated,[1] and let t_1, and t_2 be the temperatures of the water at the moment when the circuit was first made and finally broken. These temperatures are to be inferred from the observations made before and after the experiment (see ¶ 93, 2). Since the average temperature of the water agrees with that of the room, no allowance need be made for cooling in the mean time (¶ 93, 3). The quantity of heat, H, generated by the electrical current is therefore —

$$H = (w + c) \times (t_2 - t_1).$$

Now let T be the time in seconds during which this heat was generated; then the average rate at which the heat was generated must have been $\frac{H}{T}$ units per second. Since 1 unit of heat per second corresponds to a power of 4.166 watts (§ 15), the power, P, spent by the electrical current, in watts, is —

$$P = 4.166 \frac{H}{T} = \frac{4.166\,(w + c) \quad (t_2 - t_1)}{T}. \qquad \text{I.}$$

We now calculate the average current, C, in ampères, from the angles of deflection (a) averaged as in ¶ 206, and from the reduction factor of the galvanometer, I, already determined (Exps. 78–81) by the formula —

$$C = I \tan a. \qquad \text{II.}$$

We have finally, by Joule's Law (§ 136) for the resistance, R, of the conductor in ohms —

$$R = \frac{P}{C^2}. \qquad \text{III.}$$

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[1] If the calorimeter is of brass, its thermal capacity is .094 W. nearly. To this should be added about 0.5 units for the thermal capacity of the thermometer and stirrer. See ¶ 90 (2).

If the experiment were varied so as to make the current just 1 ampère, then, since $C = I$, R would be equal to P. This is in accordance with the definition of resistance (§ 136). The student should bear in mind that the resistance of a conductor in ohms is nothing more or less than the power in watts required to maintain in that conductor a current of 1 ampère.

EXPERIMENT LXXXVI.

COMPARISON OF RESISTANCES.

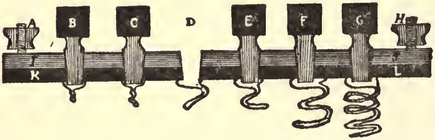

Fig. 240.

¶ 214. **Construction of a Rheostat.** — A rheostat may be constructed as in Fig. 240. A series of brass blocks (IJ) is firmly attached to a plate of ebonite (KL), which is a non-conductor of electricity. The brass blocks are connected by coils of German-silver wire, which should be well insulated with silk. Each wire should be doubled in the middle (see Fig. 240), and the double wire should be coiled up or wound on a bobbin. The equal and opposite currents in any part of the coil thus neutralize each other as far as external magnetic effects are concerned.[1] Brass plugs *B*, *C*, &c., are fitted into hollows between the blocks, so as to make good electrical connections. When all the plugs are in place, a current flowing through the blocks in series from the binding-post *A* to the binding-post *H*, should meet with a hardly appreciable resistance. If, however, one of the plugs (as *D*) is removed, the current is obliged to pass through one of the coils. It meets therefore, with a certain electrical resistance.

The resistance of the first coil in the series is usually 1 ohm (§ 20); that of the second is 2 ohms; the third and fourth are either 2 and 5 or 3 and 4 ohms. It is thus possible, by taking out one or more plugs at the same time, to introduce resistances from 1 to 10 ohms into the path of a current. The series of resistances may be extended by adding three new coils of 20, 20, and 50 ohms' resistance. With seven coils, we may thus obtain any resistance from 1 to 100 ohms. With three more coils of 200, 200, and 500 ohms resistance, we may extend the limit to 1000 ohms. With additional coils of 0.1, 0.2, 0.2, and 0.5 ohms, the resistance may be adjusted to a tenth of an ohm, &c. For convenience, extra coils of 1, 10, 100, and 1000 ohms are usually provided. The same results may be obtained by the series 1, 2, 3, 4, 10, 20, 30, &c. The line of resistances is usually bent, as in Fig. 241, so as to occupy as little space as possible. Connections with the two ends of the series are made by means of the binding-posts *a* and *d*. It is convenient for many purposes to include an entirely separate line of resistances, *befc*, in the arrangement. In the first part of this experiment the inner line will not be required. It should therefore be entirely disconnected from the outer line by the removal of the plugs which join the two lines together.

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[1] The effects of "self induction" should also be to a great extent eliminated by this method of winding the coils.

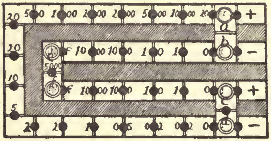

Fig. 241.

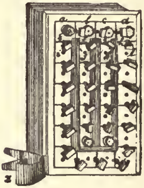

Fig. 242.

Both series of resistances are usually packed in a box (Fig. 242), variously called a "box of coils," a "resistance box," or simply a "rheostat."

¶ 215. **Determination of Resistances by the Method of Substitution.** — To find the electrical resistance of any conductor, as for instance the coil of the dyna-

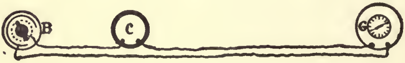

Fig. 243.

mometer employed in the last experiment, the coil (*C*, Fig. 243) is to be connected in series with a battery (*B*) and a tangent galvanometer (*G*). The deflection of the galvanometer is to be carefully observed. The dynamometer is now to be disconnected, and in its place a rheostat, *R* (Fig. 244), is to be introduced into the circuit by means of the binding-posts *c* and *d*. The plugs connecting the inner and outer lines of resistance are to be removed, so that the current can circulate only through the outer line. The plugs along this line should all be driven lightly into place, and turned round in their sockets, so as to make good electrical connections. Enough plugs are now to be removed to reduce the deflection of the galvanometer to its former magnitude.

The resistance in ohms brought into play by the removal of each plug is indicated by the number opposite its socket (Fig. 241). If the first resistance tried is too small, that is, if it fails to reduce the current sufficiently, one about twice as great is tried; if the first resistance is too large, we try one about half as great. In fact we use with a set of resistances the same method of approximation as with a set of weights (¶ 2).

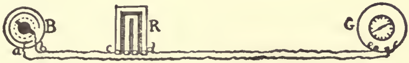

FIG. 244.

In the process of trying the several resistances, the current from the battery is liable to change. It is well, therefore, to replace the dynamometer in the circuit, and having observed the galvanometer, to substitute immediately the box of resistances (as previously adjusted) for the dynamometer. When two conductors can be thus substituted one for the other in an electrical circuit without affecting the current, their electrical resistances are evidently equal according to the general principle of substitution (see § 43). We have only, therefore, to add together the resistances of those coils in the box through which the current flows, in order to find the resistance of the dynamometer.

To save time in making connections, the terminals of the coil *C* may be carried to the binding-posts *a* and *e* of the rheostat (Fig. 245). One of the battery wires is then carried to *d*, the other to the galvanometer *G*, and back to *f*. Plugs connecting *b* with *c*, *b* with *e*, and *c* with *f*, are to be removed; the others are to remain. The binding-posts *e* and *f* are thus insulated from the rest of the instrument. The battery current then flows from *d* to *a* through the outer line of resistances, then from *a* to *e* through the coil *C*, then through *f* to the galvanometer *G* and back to the battery. If *b* and *c* be now connected by the insertion of a plug, the current will flow directly from *d* to *a*, and thus the rheostat resistance will be "cut out of the circuit." If the plug connecting *b* and *c* be removed and inserted between *b* and *e*, the current, after flowing through the outer line of resistances,

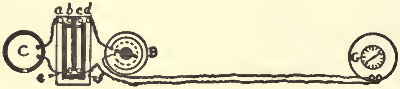

Fig. 245.

will make a short circuit from *b* to *e*, instead of passing through the coil *C*. The coil will therefore be "cut out of the circuit." By moving a single plug, accordingly, from one place to another, the rheostat may be substituted in the circuit for the dynamometer, and *vice versa*. The accuracy of the units indicated by the box of resistances may be provisionally taken for granted.

¶ 216. **Determination of Resistances by the Method of Interchange.**—A battery, *B*, (Fig. 246), is to be connected with a coil, *C*, of unknown resistance, and with a rheostat, *R*, of variable resistance in multiple arc (§ 140). The wires from the coil and from the rheostat are to be carried back to the battery, each through one half of a differential galvanometer, *G G*. The resistance of the rheostat is to be adjusted if possible, by the removal of plugs, so that the deflection of the galvanometer may be reduced to zero. Since this occurs when the currents through the two halves of the galvanometer are equal, the total resistance in the two branches of the circuit containing *C* and *R* must be equal. Assuming therefore that the two halves of the galvanometer and the connecting wires have equal resistances, the resistance of the coil *C* must be equal to that of the rheostat *R*.

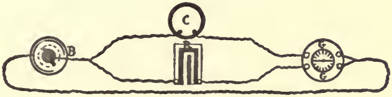

FIG. 246.

To make sure that the two halves of the galvanometer are exactly alike, the positions of the coil (C) and rheostat (R) should now be interchanged, and the resistance of the rheostat readjusted if necessary.

In the absence of a set of resistances by which the rheostat may be adjusted within, let us say, $\frac{1}{10}$ of an ohm, two adjustments must be made. In one, the resistance (R_1) of the rheostat will be too small, and the galvanometer will be deflected $x°$ in one direction. In the other adjustment the resistance (R_2) of the rheostat will be too great, and the galvanometer will be deflected $y°$ in the opposite direction.

The resistance (R) sought can evidently be found by the ordinary method of interpolation (§ 41, ¶ 26), that is —

$$R = R_1 + \frac{x}{x+y}(R_2 - R_1), \text{ nearly.}$$

In the absence of a differential galvanometer, the student should make by the method of substitution (¶ 215) as many determinations of resistance as time will allow. Other methods of comparison will be considered in experiments which follow.

EXPERIMENT LXXXVII.

WHEATSTONE'S BRIDGE.

¶ 217. **Determination of Electrical Resistances by a Wheatstone's Bridge.** — A form of Wheatstone's Bridge used by the British Association and ordinarily known as the "B. A. Bridge," is represented, with slight modifications, in Fig. 247, which gives a view of the apparatus from above. Three strips of copper, *ab*, *ce*, and *fg*, are arranged in a line on a piece of wood, with small spaces between them. A fine German-silver or platinum wire *hj*, often called the "Bridge wire"[1] is stretched over a rail 1 metre long, graduated in *mm*. The wire is soldered at both ends to corners of the strips (*ab* and *fg*), which are turned up so as to be on a level with the wire. A cross-wire is attached to a slider (*i*, Fig. 248) so that it may be made to touch the wire *hj* at any point. Binding-posts are usually added at *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *i*. The latter serves to connect any conductor (as *Gi*) with the cross-wire, and thus to make an electrical connection between it and any point of the wire *ij*.

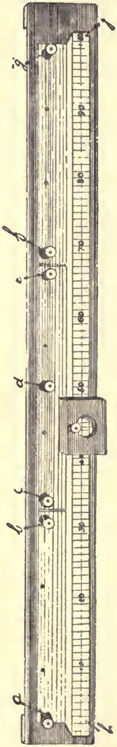

FIG. 247.

The terminals of a delicate galvanometer *G*, (see also ¶ 188, Fig. 207) are to be connected with the binding-posts *d* and *i*. The resistance coil *C*, tested in Exp. 85, is to connect *b* and *c*. Two binding-posts (*a* and *d*, Fig. 242) of the rheostat used in Exp. 86 (*R*, Fig. 248) are to be connected by thick copper wires with *e* and *f* (Fig. 248). One of the plugs is to be removed from the rheostat, so as to give a resistance of 1 ohm. The poles of a battery (*B*) are then to be connected with the binding-posts, *a* and *g*.

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[1] To avoid misconceptions arising from this name, it may be well to point out to the student at the start that the "Bridge wire" is *not* the "*Wheatstone's Bridge*" (§ 141).

The current from the battery is thus made to divide into two parts. One part flows from *a* to *d* through the coil *C*, then from *d* to *g* through the resistance *R* (or the reverse); the other part flows from *a* to *i*, through the resistance of the wire *hi*; then from *i* to *g* through the resistance of the wire *ij* (or the reverse). The resistance of all other conductors may be neglected. The galvanometer circuit forms a cross-connection or "Wheatstone's Bridge" (§ 141) between the points *d* and *i* of the parallel circuits *adg* and *aig*. The points *a*, *d*, *g*, and *i* correspond accordingly to *A*, *B*, *C*, and *D* in Fig. 18, § 141. The slider *i* is to be moved from one end of *hj* to the other until a point *i* is found having the same potential as *d* (§ 141), so that the galvanometer shows no deflection. The distances *hi* and *ij* are to be carefully measured. The poles of the battery are next to be interchanged and the experiment repeated.

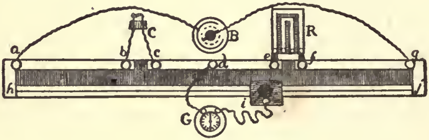

FIG. 248.

The average of the distances hi and ij is to be found. Assuming that the wire is uniform, the resistance of these portions A and B will be to each other as their lengths, hi and ij. That is —

$$\frac{A}{B} = \frac{hi}{ij}.$$

The resistance C is now calculated from the resistance R in the box of coils (1 ohm in this case) by the formula (§ 141) —

$$C = R \times \frac{hi}{ij}. \qquad \text{I.}$$

The experiment is to be repeated with the places of C and R interchanged. In this case the formula will become —

$$C = R \times \frac{ij}{hi}. \qquad \text{II.}$$

By removing from the box of coils different plugs, other measurements of the resistance C may be made. The student should satisfy himself that with various values of R, the same value of C is always obtained. The most accurate value is usually that which is found when R is nearly equal to C.

If the value of C thus determined differs by more than 10 % from that found in the last experiment, the latter should be repeated. By this means gross errors in the box of coils may be found out. It should be remembered that the British Association Unit which is copied in many boxes of coils is only about 987 thousandths of a true ohm.

EXPERIMENT LXXXVIII.

SPECIFIC RESISTANCE.

¶ 218. **Specific Resistance.** — The specific electrical resistance of a given material may be defined as the resistance of a conductor made of that material, 1 *cm.* long and 1 *sq. cm.* in cross-section. In the practical units of the volt-ohm-ampère series, the specific resistance, S, is equal accordingly to the electromotive force in volts (see § 138) required to maintain a current of 1 ampère between two opposite faces of a centimetre cube cut out of a given substance; or again, it is equal to the *power in watts* (see § 137) required to do the same thing. The power required to maintain a current of 1 ampère through L centimetre-cubes of the substance, arranged in series, so that the same current traverses each, is obviously LS watts. If we place Q rows of centimetre-cubes side by side, each row containing L of the cubes, it is obvious that to maintain a current of 1 ampère in each row will require LS watts; hence the total power required for all the rows will be QLS watts.

Since each row is traversed by a current of 1 ampère, the compound conductor, consisting of Q rows, must carry a current of Q ampères.

The resistance of this conductor may now be calculated by Joule's Law ($P = C^2 R$, see § 136);

for substituting QLS for P, and Q for C, we have —

$$R = \frac{P}{C^2} = \frac{QLS}{Q^2} = \frac{LS}{Q}. \qquad \text{I.}$$

We notice that in the formula L represents the length and Q the cross-section of the compound conductor. The resistance of any conductor is accordingly proportional to its length, and inversely as its cross-section. To find it, we multiply the specific resistance by the length and divide the product by the cross-section. Obviously, specific resistances of different materials are important factors in calculations relating to electrical resistance.

To calculate specific resistance (S), we must first find the actual resistance (R) of a conductor of known length (L) and cross-section (Q); we then have, from I., —

$$S = \frac{RQ}{L}. \qquad \text{II.}$$

It will be found convenient to express the result in terms of microhms (§ 2) instead of ohms. This is done by moving the decimal point six places to the right (*i. e.*, multiplying by 1,000,000).

¶ 219. **Determination of Specific Resistance.** — A fine German-silver wire (not insulated), about 1 metre long, is soldered (near *a* and *b*, Fig. 249) to two copper strips. These strips are to be so thick that their electrical resistance may be neglected. They are to be scraped bright (¶ 193, 11), and connected with the binding-posts *b* and *c* of a Wheatstone's bridge apparatus, in place of the coil used in the last experiment (see Fig. 248, ¶ 217). To prevent the wire from crossing itself at any point, it may be looped round a glass jar *a* (Fig. 249). The resistance (R) of the wire is to be found as in the last experiment.

The wire is now to be straightened, and the distance *between* the copper strips accurately determined. This gives the length (L) of the conductor spoken of in the last section. The diameter (d) of the wire is to be measured at let us say ten different points with a micrometer gauge (¶ 50, II.), and the results averaged. The cross-section (Q) of the wire is then calculated by the ordinary formula —

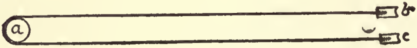

FIG. 249.

$$Q = \tfrac{1}{4} \pi d^2.$$

The specific resistance of the German silver of which the wire is composed is finally to be calculated by formula II. of the last section.

The experiment may be repeated with wires of different lengths, diameters and materials.

EXPERIMENT LXXXIX.

THOMSON'S METHOD.

¶ 220. **Determination of the Resistance of a Galvanometer by Thomson's Method.** — The terminals of a galvanometer, *G* (Fig. 250), and of a rheostat, *R*, are to be connected with a Wheatstone's Bridge apparatus in the same manner as any other resistances would be connected, when it is desired to compare them together (see Exp. 87). A battery, *B*, is also to be connected in the same manner. Instead, however, of putting a second galvanometer in the circuit *di*, to tell when the current in that circuit is reduced to zero, a simple key, *K*, is placed there.

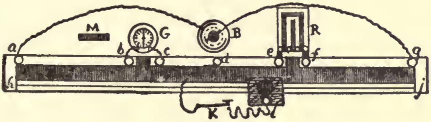

FIG. 250.

The galvanometer needle will probably be strongly deflected by the current passing through the instrument. It must be brought back nearly to zero by a powerful magnet, M, properly placed. If the battery is too strong for the magnet, a weaker battery may be substituted, or the same result may be obtained by connecting the poles of the battery with a cross-wire or shunt of sufficiently low resistance. The key is now to be closed. If the effect is to increase the deflection of the needle, the slider (i) is to be moved toward that end of the "Bridge wire" (hj) nearest the galvanometer. If the effect is to diminish the deflection, the slider is to be moved toward the rheostat. Finally a point (i) is found where the closing of the key has no effect upon the galvanometer. The resistance of the latter is then calculated as in the last experiment.

The experiment is to be repeated with a rheostat resistance as nearly as possible equal to that of the galvanometer. The current should be reversed, and the resistances interchanged as in Experiment 87.

The resistance of the galvanometer is to be calculated by one of the formulæ of ¶ 217.

¶ 221. **Explanation of Thomson's Method.** — Thomson's method of measuring the resistance of a galvanometer depends upon the fact that when the circuit *di* (Fig. 250) is closed through K, more or less current will ordinarily pass from i to d, or the reverse.

The electrical potential (§ 139) of the point d will therefore be affected, just as the pressure at a given point in a water pipe would be affected by connecting that point with one in another pipe where the pressure was different. Since the current from a to d depends (according to Ohm's Law, § 138) upon the difference of potential between those points, it is evident that if a retains the same potential as before, any change in the potential at d must affect the current. The deflection of the galvanometer is accordingly increased or diminished. The object of nearly neutralizing the deflection is that any change in it may be made perceptible; for if the needle were already deflected for instance 89°, since 90° is the maximum possible deflection, it would be hard to detect an increase in the current. We have seen that the electrical potential at *d* is changed when it is connected with a point *c* at a different potential; obviously if *d* and *i* are at the *same potential*, there will be no change in the potential of *d*, and hence no change in the deflection of the galvanometer. The student should note that we may find a point *i*, having the same potential as a point *d*, either (1) by observing the deflection of a galvanometer in the circuit *di* (see Exp. 87), or (2) by observing the *change* in the deflection of a galvanometer in any other branch of the compound circuit.

The chief difficulty in this experiment lies in the arrangement of a permanent magnet so as to neutralize the deflection of a galvanometer needle without destroying temporarily the sensitiveness of the instrument. The advantage of this method, aside from its theoretical interest, is chiefly in cases where it is impossible to obtain a second galvanometer sufficiently sensitive to measure the resistance of the first.

EXPERIMENT XC.

MANCE'S METHOD.

¶ 222. **Determination of the Internal Resistance of a Battery by Mance's Method.** — A rheostat (*R*, Fig. 251) and a galvanometer (*G*) are to be connected with a Wheatstone's Bridge apparatus as in Experiment 87; and a battery cell (*B*) is to be put in place of the unknown resistance (*C*, Fig. 248). Instead, however, of placing a second battery in the circuit *ag*, a simple key (*K*) is put there.

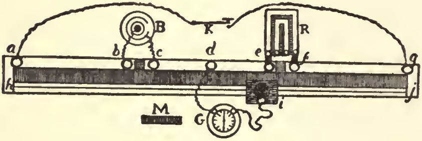

Fig. 251.

The needle of the galvanometer will probably be strongly deflected by the current passing from *d* to *i*, or the reverse. As in the last experiment, this deflection must be nearly reduced to zero, by bringing a powerful magnet (*M*) near the galvanometer. A shunt may be introduced if necessary between the terminals of the galvanometer (see ¶ 193, 2). The key is now to be closed. If the deflection of the galvanometer is increased, the slider (*i*) is to be moved toward the battery. If the deflection is diminished, it should be moved toward the rheostat. The change in the position of the slider will probably throw the galvanometer and magnet out of adjustment. The position of the magnet must therefore be changed. After a series of trials the slider may be placed at a point *i*, where no *sudden* effect is produced upon the galvanometer by closing the key.

If the galvanometer is affected one way when the key is first closed, then the other way, the first effect is the one by which the adjustment of the slider is to be made.

The experiment is to be repeated with a resistance in the rheostat as nearly as possible equal to that of the battery; but the methods of reversal and interchange employed in Exp. 87 will hardly be justified by the accuracy of the experiment. The resistance of the battery is to be calculated by one of the formulæ of ¶ 217.

¶ 222 a. **Explanation of Mance's Method.**— The effect in Mance's method of the battery current upon the galvanometer has generally to be diminished by shunting the galvanometer. The opposite difficulty however, sometimes arises. When it is desired to measure the resistance of a battery composed of two nearly equal cells, opposed to one another, the current from these cells may be insufficient to affect the galvanometer. In this case an auxiliary battery must be introduced into the circuit *akg*. We will first suppose that such an auxiliary battery is employed. If the two cells of which the resistance is to be measured *exactly* neutralize each other, the case differs from that of an ordinary Wheatstone's Bridge only in the nature of the resistance which is to be measured. The theory is therefore the same.

If, however, one of the two cells is stronger than the other, an allowance must be made for the current which flows from the battery (B) through the galvanometer, whether the auxiliary battery is connected or not. This is done by neutralizing the deflection of the galvanometer due to the battery B.

The fundamental principle upon which Mance's method depends is that two batteries in any system of conductors, however complicated, produce each the same effect as if the other were not present. The current in any part of the circuit is in fact the algebraic sum of the two currents which the batteries would separately produce. We have seen that a battery in the circuit *akg* affects a galvanometer in the circuit *di*, unless the resistances *ai* and *ij* are proportional to *ad* and *dg* respectively. If a current already exists in the galvanometer a *change* in that current must be produced by a battery in the circuit *aky*, unless the proportion above is fulfilled.

Let us now suppose that the battery in the circuit *akg* is just strong enough to neutralize the current from the battery B, which would naturally flow through the circuit *akg*. Then the effect of introducing this battery into the circuit may be simply to arrest the current in *akg*. The same effect is produced by breaking the circuit by means of the key K. Evidently the act of opening or closing the key in a circuit is equivalent to connecting or disconnecting a battery of considerable strength.

When the circuit is made the resistance between the poles of the battery is much less than when the circuit is broken. The result is an increased current from the battery, and in a very short time a change in its electromotive force. The observations should, therefore, be taken the moment that the circuit is closed. The galvanometer needle sometimes first jumps in one direction, then slowly changes to the other direction. The slow movement in the needle may be explained as the result of a gradual change in the electromotive force of the battery. The first effect indicates which of the resistances is too great or too small.

The chief advantage of Mance's method is that it enables us to measure the resistance of batteries at a given instant while furnishing a current. Concordant results must not be expected between Mance's and other methods. It is now thought that there is something not yet understood in the nature of battery resistances which causes these resistances to appear to be greater or less according to the manner in which they are determined.

EXPERIMENT XCI.

USE OF A SHUNT.

¶ 223. **Determination of the Resistance of a Galvanometer by means of a Shunt.** — I. Two tangent galvanometers (*ab* and *gh*, Fig. 252) already employed in Exp. 79, are to be set up in the same places as in that experiment, and connected in series with a battery (B) capable of causing deflections of from 50° to 60°. The connecting wires *bcdeg* and *afh* are to be made bare at a point between the two galvanometers and at a point (e) between the galvanometer (gh) and the battery. The wires are to be clamped at these points by the binding-posts of a rheostat (R). All the plugs are now to be put into their places. The galvanometer *gh* will then be short circuited through the rheostat (R). The deflection of the galvanometer should accordingly fall to 0°. If it does not, the plugs in the rheostat should be turned round in their sockets with light pressure until at least a minimum deflection is obtained.[1]

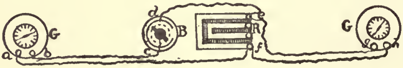

Fig. 252.

When plugs are removed from the box of coils, a part only of the current will flow through the rheostat. The galvanometer (gh) will then be deflected. Plugs are to be removed from the box until the deflection of the galvanometer (gh) reaches about 30° or a little more than half the deflection of *ab*. The resistance of the rheostat is to be noted, and the deflections of the two galvanometers are to be simultaneously determined as in Exp. 82. This method is applicable to galvanometers of low resistance. The results are to be reduced by ¶ 224, I., formula (5).

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[1] The plugs should be carefully cleaned if necessary by rubbing them with paper.

II. Instead of the galvanometer *ab*, a second rheostat resistance may be introduced into the circuit *edcbaf*. The value of this resistance is to be noted. The deflections of the galvanometer *gh* must be observed (as in I.) with and without the shunt *ef*. The resistance of the shunt must also be noted.

This method requires a constant battery (see Exp. 84), with an internal resistance which is either known (see Exps. 92 and 93) or so small that it may be neglected in comparison with the resistance in the circuit *edcbaf*. The method is used in practice only in the case of high-resistance galvanometers. On account of the extreme sensitiveness of such instruments, the current from an ordinary voltaic cell must be reduced by the use of a very large resistance in the circuit *edcbaf*. In comparison with this resistance, that of the voltaic cell may usually be neglected. The resistance of the shunt should be such that when connections are made through it, the deflection of the galvanometer may be about half as great as when these connections are broken. The results are to be reduced by ¶ 224, II., formula (12).

¶ 224. **Calculations of Resistance depending upon the Use of a Shunt.** — I. If I and i are the reduction factors of the two galvanometers, A and a their deflections, then since the whole current, C, passes through the first galvanometer (*ab*, Fig. 252), it must be given by the equation (see formula 7, ¶ 199) —

$$C = I \tan A. \qquad (1)$$

Only a portion (c) of this current passes through the second galvanometer (gh); this portion is —

$$c = i \tan a. \qquad (2)$$

The remainder (c') of the current flows through the rheostat. Evidently —

$$c' = C - c = I \tan A - i \tan a. \qquad (3)$$

Now the current (c) through the galvanometer (gh) must be to that (c') through the shunt inversely as the resistances (let us say G and S) in question (§ 140). That is —

$$c : c' :: S : G. \qquad (4)$$

The resistance of the galvanometer (G) may therefore be found by the formula —

$$G = \frac{c' S}{c} = S \frac{I \tan A - i \tan a}{i \tan a}. \qquad (5)$$

It should be remembered that the resistance of the galvanometer (gh, Fig. 252), calculated by this formula, includes that of the wires, eg and fh, connecting it with the rheostat. The result is rendered inaccurate by any bad connection within the rheostat. A minimum deflection of 1° in the galvanometer (gh), produced with all the plugs in place in the rheostat (R), indicates an under estimate of both the galvanometer and rheostat resistances not far from 1 or 2 %.

II. If E is the electromotive force of the battery (B, Fig. 252), R the resistance in the circuit *edcbaf* (including strictly the internal resistance of the battery), and if G is the resistance of the galvanometer, the current, C, produced (when the connection between e and f is broken) must be (see § 138) —

$$C = \frac{E}{R + G}. \qquad (1)$$

If now a connection is made between e and f through a shunt of the resistance S, so that the current flows partly through G and partly through S, the resistance (r) of this multiple circuit will be (solving the equation in § 140) —

$$r = \frac{GS}{G + S}. \qquad (2)$$

The current C' now becomes —

$$C' = \frac{E}{R + r}, \qquad (3)$$

or, substituting the value of r and reducing, —

$$C' = \frac{E\,(G + S)}{RG + RS + GS}. \qquad (4)$$

The portion (c) of this current which flows through the galvanometer is to the whole current (C') as S is to $G + S$ (§ 140); that is —

$$c = C'\,\frac{S}{G + S}. \qquad (5)$$

Substituting the value of C' from (4) we have —

$$c = \frac{E\,(G + S)}{RG + RS + GS} \times \frac{S}{G + S} \text{ or}$$

$$c = \frac{ES}{RG + RS + GS}; \qquad (6)$$

hence $$E = \frac{cRG + cRS + cGS}{S}. \qquad (7)$$

But from (1) $E = CR + CG$;

hence $$\frac{cRG + cRS + cGS}{S} = CR + CG, \qquad (8)$$

$$cRG + cRS + cGS = CRS + CGS, \qquad (9)$$

$$cRG + cGS - CGS = CRS - cRS, \qquad (10)$$

and $$G\,(cR + cS - CS) = RS\,(C - c), \qquad (11)$$

whence, finally, $$G = \frac{RS\,(C - c)}{cR + cS - CS}. \qquad (12)$$

In the use of this formula it is necessary to know only the relative values of the currents C and c. With nearly all instruments, when the deflections are small, the currents are proportional to these deflections. We may accordingly substitute the deflections produced in such cases for the currents which they represent.

EXPERIMENT XCII.

OHM'S METHOD.

¶ 225. **Determination of the Resistance of a Battery by Ohm's Method.** — A tangent galvanometer (*G*, Fig. 253) and a rheostat (*R*) are to be connected in series by the wires *bc*, *de*, and *af*, with a Daniell cell (*B*) capable of deflecting the galvanometer needle 50° or 60° when all the plugs of the rheostat are in their places. The deflection of the galvanometer is to be accurately observed. The 1-ohm plug is now to be removed from the rheostat, and the deflection again noted. The resistance of the rheostat is then gradually increased until the deflection of the galvanometer is reduced to less than half of its original magnitude. In each case, the deflection is to be carefully observed, and the resistance noted.

The connections at *b* and *f* being now interchanged (¶ 193, 9) so that the direction of the current through the galvanometer is reversed, the experiment is to be repeated. If any differences are observed in the deflections corresponding to a given resistance, the mean angle of deflection is to be calculated in each case.

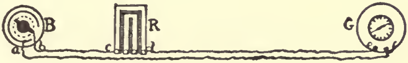

FIG. 253.

If a_1 and a_2 are the mean angles of deflection in any two cases, R_1 and R_2 the corresponding rheostat resistances, C_1 and C_2 the currents through the galvanometer, I the reduction factor of the galvanometer (Exps. 78, 80, 81), B the resistance of the battery, galvanometer, and connecting wires, then we have (see ¶ 199, 7) —

$$C_1 = I \tan a_1 \quad (1); \quad C_2 = I \tan a_2. \qquad (2)$$

Now by Ohm's law (§ 138) these currents are inversely as the corresponding resistances, that is —

$$C_1 : C_2 :: R_2 + B : R_1 + B, \tag{3}$$

hence we find —

$$\frac{R_2 + B}{R_1 + B} = \frac{C_1}{C_2}, \tag{4}$$

$$R_1\ C_1 + BC_1 = R_2\ C_2 + BC_2, \tag{5}$$

$$BC_1 - BC_2 = R_2\ C_2 - R_1\ C_1, \tag{6}$$

$$B\ (C_1 - C_2) = R_2\ C_2 - R_1\ C_1, \tag{7}$$

$$B = \frac{R_2\ C_2 - R_1\ C_1}{C_1 - C_2}, \tag{8}$$

and finally, substituting the value of C_1 and C_2, and cancelling I, we have —

$$B = \frac{R_2 \tan a_2 - R_1 \tan a_1}{\tan a_1 - \tan a_2}. \tag{9}$$

The student may thus calculate several values of B. The best value for R_1 is 0; that is, we obtain the most accurate results by utilizing the observation of the galvanometer when all the plugs are in place. Evidently if $R_1 = 0$, the value of B becomes simply

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2}. \tag{10}$$

The best value for R_2 is one nearly equal to B. The simplest way to find this value is to calculate the value of B from any two of the observations. It must be remembered that the battery resistance thus calculated includes that of the galvanometer and connecting wires. Having found the resistance of the galvanometer, &c. from the last experiment, we may find by subtraction the *internal* resistance of the battery. The results with a tolerably constant battery should agree with those obtained by Mance's Method (Exp. 90) within 5 or 10 %.

The calculation of the electromotive force of a battery from the results of Ohm's Method will be considered in ¶ 230. It may be remarked that if this electromotive force is not constant, formula (3) is not justified. In this case the succeeding formulæ which depend upon (3) may give false or even absurd results.

EXPERIMENT XCIII.

BEETZ' METHOD.

¶ 226. **Explanation of Beetz' Method.** — In Beetz' method two batteries, B' and B'' (Fig. 254) are placed in the same circuit (*abcda*) but so as to be opposed to each other; and the circuit is divided into two lobes, like a figure 8, by means of a wire *ac*, acting as a shunt to both batteries. A known resistance R' is placed between *b* and *c*; another known resistance (R) is introduced between *a* and *c*; a delicate

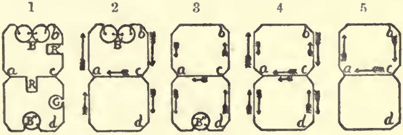

FIG. 254.

galvanometer (G) is placed between c and d. We will suppose that the two positive poles of the batteries are connected at c.

Let us now consider what effect the battery B' would produce if B'' were not acting. The current descending in the branch bc would divide into two parts (Fig. 254, 2); one flowing directly from c to a, the other indirectly from c to a through d. These two parts would unite at a, and thence return to the battery.

Let us next consider what effect B'' would produce if B' were not acting. The current ascending in dc (Fig. 254, 3) would divide into two parts; one flowing directly from c to a, the other indirectly from c to a through b. Both parts uniting at a would return to the battery.

When both batteries act together, each may be considered to produce the same effect as if the other were not acting. The result is represented in Fig. 254, 4. We notice that in the diagrams the *portion* of the current from B' which flows through d is as great as the *whole* current from B''. To produce this effect it is evident that the battery B' must be stronger than B''. It is also evident that two equal and opposite currents through d must neutralize each other; hence the result of combining two batteries as in Fig. 254 may be such as is represented in Fig. 254, 5; namely, a current entirely confined to the circuit bc, containing the stronger battery, *no current whatever flowing through the weaker battery.*

In practice we employ a battery, B', *more than suffi-*

cient to reverse B''; then we weaken the current which it sends through the circuit d, either by increasing the resistance R', so that the whole current from B' is reduced, or by diminishing the resistance R, so that a greater portion of the current may flow directly from c to a, without passing through the battery B''. The use of the galvanometer, G, is simply to tell when an exact balance has been established between the two opposing currents through d (see Fig. 254, 4). No current is then indicated by the galvanometer.

It is possible to calculate by Ohm's Law (§ 138) and by the principle of divided circuits (§ 140) the magnitude of each of the currents represented in Fig. 254, 4, and thus to find under what conditions the currents through d are equal and opposite. The expressions become, however, more or less complicated. The final solution, which is simple, may be obtained much more easily by the method which follows.

¶ 227. **Principle of Electromotive Forces in Equilibrium.** — Let E' be the electromotive force, and B' the resistance of the first battery; let E'' be the electromotive force of the second battery (B''), and let C be the current through the rheostat R. Then if, according to the diagram (Fig. 254, 5) the current through B'' has been reduced to zero, the current C, having no choice of circuits must flow through B' and R' as well as through R. The result is the same as if the circuit through B'' did not exist. We have accordingly an electromotive force E', causing a current C through a total resistance $R + B' + R'$. Hence, by Ohm's Law (§ 138), —

$$E' = C\,(R + B' + R'). \qquad (1)$$

The power of the battery is spent in heating the several resistances R, B', and R'. We need to consider only the power (P) spent in heating the resistance R. We have (see § 136) —

$$P = C^2\,R. \qquad (2)$$

The ratio of this power (P) to the current (C) determines that part (E) of the whole electromotive force (E') which is required to maintain the current (C) through the resistance (R) in question. Since in passing through the resistance R the loss of potential is E, we have (see §§ 137, 138, and 139) —

$$E = \frac{P}{C} = \frac{C^2\,R}{C} = CR. \qquad (3)$$

The power spent by the battery B'' upon a small current C'' flowing through it in the ordinary direction (from a to c) will be $C''\,E''$ (§ 137); but the power required to take electricity from a point a to a point c, where the electrical potential is higher than at a by the amount E, is $C''\,E$. Evidently such a current through the battery can exist only on condition that E'' is greater than E.

On the other hand, a current C'' flowing from c to a would represent an expenditure of power equal to $C''\,E$. The power required to drive the current backward through the battery B'' is, however, $C''\,E''$.

Evidently a reversed current can exist only if E is greater than E''. It follows that if E and E'' are equal, the current through B'' will be reduced to zero. It is evident, conversely, that if the galvanometer in the diagram (Fig. 254, 1) shows no deflection, E and E'' must be equal; that is (from 3), —

$$E'' = CR; \tag{4}$$

from which we find —

$$C = \frac{E''}{R}, \tag{5}$$

a formula by which we may calculate the current from a battery (B') which, flowing through a known resistance, R, neutralizes a known electromotive force, E''.

¶ 228. **Calculation of Battery Resistances in Beetz' Method.** — For the determination of the resistance of a battery by Beetz' method, two experiments are necessary. Let r_1 and r_1' be the values of R and R' (¶ 226) in the first experiment, and let r_2 and r_2' be the corresponding values in the second experiment. Then from ¶ 227 we have, dividing (1) by (4), —

$$\frac{E'}{E''} = \frac{B + r_1 + r_1'}{r_1}, \tag{1}$$

and

$$\frac{E'}{E''} = \frac{B + r_2 + r_2'}{r_2}. \tag{2}$$

Assuming that the proportion between E' and E'' is the same in both experiments, we have, equating (1) and (2), —

$$\frac{B + r_1 + r_1'}{r_1} = \frac{B + r_2 + r_2'}{r_2} \qquad (3)$$

$$Br_2 + r_1\, r_2 + r_1'\, r_2 = Br_1 + r_1\, r_2 + r_1\, r_2' \quad (4)$$

$$Br_2 - Br_1 = r_1\, r_2' - r_1'\, r_2 \qquad (5)$$

$$B = \frac{r_1\, r_2' - r_1'\, r_2}{r_2 - r_1}. \qquad (6)$$

The same result may be obtained from formula (8), ¶ 225, namely, —

$$B = \frac{R_2\, C_2 - R_1\, C_1}{C_1 - C_2}, \qquad (7)$$

by substituting for the total external resistances R_1 and R_2 their values, $r_1 + r_1'$ and $r_2 + r_2'$ respectively, and also substituting for the two corresponding currents C_1 and C_2 their values (from ¶ 227, formula 5) $\frac{E''}{r_1}$ and $\frac{E''}{r_2}$ respectively. The factor E'' is cancelled in the reduction.

Beetz' method differs from Ohm's method chiefly in the manner in which we estimate the relative strength of two currents. In Ohm's method the ratio between the currents is determined by the angles of deflection produced in a tangent galvanometer. In Beetz' method, it is determined by the resistance between the poles of a constant battery, enabling the current to neutralize the effect of that battery. Beetz' method is essentially a null method (§ 42).

Beetz' method may be used not only to measure the resistance of a battery (see 6), but also, when that resistance has been found, to determine the relative magnitude[1] of two electromotive forces (see 1 and 2, also ¶ 230, 8).

When the electromotive force of a battery is known, it furnishes us with the means of measuring currents with great precision (see formula 5, ¶ 227).

¶ 229. **Determination of Battery Resistances by Beetz' Method.** — The copper or positive pole (*P*, Fig. 255) of a battery (*B*), consisting of two Daniell cells in series, is to be connected by a wire (*PKK'P'*) with the positive pole (*P'*) of a weaker battery (*B'*). The circuit is to be completed between the negative poles (*N'* and *N*) of the batteries through a delicate galvanometer (*G*) provided with a shunt (*S*) to prevent it from being injured by the battery currents (¶ 193, 2) and through the *inner* line of resistances, *bc*, of a box of coils. The inner and outer lines, *bc* and *da*, are to be connected with a plug between *c* and *d*, but separated at *a* and *b* throughout the experiment. The wire *PKK'P'* is to be made bare at *a* and connected at that point with the binding-post of

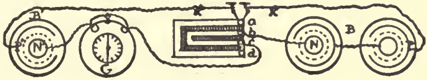

Fig. 255.

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[1] If a tangent galvanometer be introduced into the circuit of the stronger battery (*B'*), for instance between *a* and *b* (Fig. 254), so that the current *C* becomes known, we may calculate also the absolute values of the electromotive forces by formulæ (1) and (4) of ¶ 227. This important modification of Beetz' method is due to Poggendorff. See ¶ 230, 3, and Exp. 99.

the outer line of resistances. Keys (*K* and *K′*) are to be placed one on each side of *a*. When all the plugs are in place, and the keys closed, the circuit of the battery (*B*) is completed through the lines of resistance *bc* and *da*, the course of the current being *PKadcbN*. The circuit of *B′* is also completed through the outer line *da*, thus: *P′K′adcGN′*. The student should note the direction in which the galvanometer is deflected.

When the connection between *a* and *d* is broken by removing the "infinity plug,"[1] both of the circuits named above are interrupted. If the keys *K* and *K′* are closed, the batteries will be opposed to one another. Neither battery can furnish a current unless it is strong enough to force it backward against the other battery. If the battery *B* is stronger than *B′*, the current will follow the course *PKaK′P′N′GcbN*. Since the current in *B′* is reversed, the galvanometer will be deflected in the opposite direction. The student should make sure that this is the case. If it is not, there is probably some error in the connections, which must be corrected.

The infinity plug is now to be returned to its place, and other plugs removed between *a* and *d*.

It will be seen that when the resistance of the outer line *ad*, common to the two battery circuits, is very small, the galvanometer is deflected one way; when the resistance is very large the galvanometer is deflected the other way. The next step is to find, by gradually increasing the resistance, at what point the change in the deflection takes place.

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[1] Two of the brass blocks in each chain of resistances should have no metallic connection between them, except that furnished by the plug. When the plug is removed there should be no perceptible current from one block to the other. In other words, the resistance between the blocks should be practically infinite. The plug in question is called accordingly the "infinity plug." It is usually marked ∞ or INF.

To avoid using up the batteries (¶ 193, 10), the keys K and K' should be left open, except at the moment when it is desired to test the deflection of the galvanometer. The key K in the circuit of the stronger battery is always to be closed first, then the other key, K', immediately after it. As soon as the direction of the deflection has been recognized, the keys are opened in the inverse order.[1]

If the galvanometer is deflected in the same way as when all the plugs are in place, the resistance of the outer line (*ad*) is to be increased; if it is deflected as when the connection in *ad* is broken, the resistance is to be diminished. The sensitiveness of the galvanometer may be increased if necessary by removing the shunt (S) but the student must not forget to replace the shunt before proceeding to the second part of the experiment. The resistance of the outer line (*ad*) causing the deflection of the galvanometer to disappear is to be recorded. If no such resistance can be found, the two nearest resistances should be noted, and the deflections (one in one direction, the other in the other direction) caused by each should be observed. From these results the desired resistance is to be calculated as in ¶ 216, by interpolation (§ 41).

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[1] A "double key" or other mechanical contrivance for closing two circuits one after the other will be found useful in this experiment.

So far the resistance in the inner line *bc* has been zero. This resistance is now to be increased by removing the 10-ohm plug. If the keys be closed, the galvanometer will be deflected. To reduce the deflection to zero, it will be necessary to increase the resistance of the outer line (*ad*). The resistances of both parts of the rheostat (*bc* and *ad*), causing equilibrium in the galvanometer are to be noted.

The battery resistance is to be calculated by formula 6, ¶ 228; remembering that the values of *ad* correspond to the resistances r_1 and r_2, common to the two circuits, while the values of *bc* correspond to the resistances r_1' and r_2', in the circuit of the stronger battery.

ELECTROMOTIVE FORCE.

¶ 230. **Different Methods for the Determination of Electromotive Forces.**

I. Absolute Methods. Electromotive force (see § 137) is defined as the ratio of the power spent by any source of electricity to the current which it produces. We must distinguish between methods (1–4) in which the power thus expended is absolutely measured and those (5–12) in which comparative results only are obtained.

(1) Method of Heating. The power spent by an electric current may be measured in the same way as electrical resistance (Exp. 85), by passing a current from a battery through a coil of wire surrounded with water, and calculating from the rise of temperature of the water how much energy has been spent by the current in a given length of time.[1] If the strength of the current be known, the loss of potential may be found by the general formula (§ 137) —

$$E = \frac{P}{C}.$$

Thus if a current of 2 ampères is found to heat the equivalent of 100 grams of water 15° in 1000 seconds, so that it generates $1\frac{1}{2}$ units of heat in one second, since 1 unit of heat per second is equivalent to 4.166 watts (§ 15), 1½ units per second would be equivalent to 6.249 watts, or 6.249 ÷ 2 = 3.124 watts per ampère. We know, therefore, that the difference in potential (§ 139) between the two ends of the coil of wire must be 3·124 volts. It will not do, however, to assume that this is equal to the electromotive force of the battery; for we have left out of account the heat generated by the electrical current in the connecting wires and in the interior of the battery. Unless the electrical resistance of the battery be unusually small in comparison with that of the coil, a considerable portion of the electrical energy will be thus wasted.

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[1] See Glazebrook and Shaw, Practical Physics, § 74.

At the same time that the method of heating can not in practice be employed to determine *directly* the electromotive force of a battery, it must be remembered that all determinations of electromotive force which involve a measurement of current and resistance may depend *indirectly* upon the method of heating, since this is one of the fundamental methods by which resistances are measured (Exp. 85).

(2) OHM'S METHOD. Having once determined a standard of resistance by the Method of Heating (Exp. 85), we have seen how by various methods of comparison (Exp. 86–93) the resistance of any part of an electrical circuit may be found. In Ohm's method, we find the current (C) in a simple circuit, and calculate the resistance (R) of this circuit by adding together the resistances of its separate parts.

Then, by Ohm's Law, we have for the electromotive force (E) the general equation (§ 138) —

$$E = CR.$$

Substituting in this formula the value of R, which in the absence of any resistance except that of the battery, galvanometer, and connecting wires, is given by formula 10, ¶ 225, namely —

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2},$$

and substituting also the corresponding value of C, namely, $I \tan a_1$, we have —

$$E = \frac{IR_2 \tan a_1 \tan a_2}{\tan a_1 - \tan a_2}.$$

The student may show that the same formula is obtained if we multiply the total resistance ($B + R_2$) in the second part of the experiment by the current ($C_2 = I \tan a_2$) which flows through it. The agreement of the two results must not be taken as an indication that the electromotive force is the same in both parts of the experiment, but as the necessary consequence of the formulæ of ¶ 225, in framing which we have *assumed* that the electromotive force of the battery is constant.

(3) POGGENDORFF'S METHOD. It has already been shown in Beetz' method (Exp. 93) that the current from a battery may be neutralized by meeting a counter current caused by division of a current from a more powerful battery into two parts. This is

the principle of Poggendorff's absolute method (see Exp. 99), which differs from Beetz' method simply in the fact that a tangent galvanometer is introduced into the circuit of the more powerful battery (B', Fig. 254) as a means of measuring the current (see note, ¶ 228). Given the current, C, and the resistance, R, the electromotive force (E) is calculated by the ordinary formula (§ 138) —

$$E = CR.$$

(4.) ELECTROSTATIC METHODS. The electromotive force of a powerful battery may be measured by the repulsion between two pith-balls charged by the battery under certain conditions (see ¶ 258). Electrostatic forces are also measured in absolute electrometers of various kinds (see ¶ 270). It should, however, be remembered that results obtained by such instruments are strictly in the electrostatic system. Since the relation between the electrostatic and the ordinary (electromagnetic) systems are not known with any great degree of accuracy, the use of electrometers, as far as the latter system is concerned, is practically confined to the comparison of electromotive forces (see ¶ 230, 11, also ¶ 270).

II. COMPARISON OF ELECTROMOTIVE FORCES. The absolute measurement of electromotive force is, like the absolute measurement of resistance upon which it depends, a more or less difficult problem. The *comparison* of two electromotive forces may, however, be made with a considerable degree of precision.

(5) THE VOLT-METER. Two electromotive forces may be compared by the currents separately produced by them through equal resistances. When the resistance of a battery is unknown, it is evident that this method cannot in general be applied; for the battery resistance may be a considerable part of the resistance of a circuit. In practice, few batteries have a resistance of more than 10 ohms; in fact 1 ohm would be much nearer the average battery resistance. Hence if a galvanometer has a resistance of several thousand ohms, the battery resistance may usually be disregarded. This is the principle on which volt-meters are constructed (Exps. 96 and 97).

(6) WIEDEMANN'S METHOD. In Wiedemann's Method (Exp. 94), two batteries are joined in series with a tangent galvanometer of low resistance. Whether the batteries act in the same or in opposite ways, the total resistance in the circuit is the same (see note ¶ 197). It follows, therefore, from Ohm's law (§ 138), that the current is proportional in one case to the sum, in the other case to the difference of the electromotive forces E and e; hence the sum $(E + e)$ is to the difference $(E - e)$ as the currents C and c produced, that is —

$$E + e : E - e :: C : c.$$

(7) METHOD OF OPPOSITION. Let us now suppose that N cells of the electromotive force E being opposed to N' cells of the electromotive force E' reduce the current to zero, then obviously the electromotive force $NE = N'E'$; or, $E' : E :: N : N'$.

This is a fundamental method of comparing electromotive forces, the usefulness of which is limited only by the difficulty of obtaining enough cells of each kind to make an exact balance. We note that, in this method, we compare the electromotive forces of two batteries when at rest, and not (as in previous methods) when in action. The method of opposition is essentially a "null method" (§ 42) for the comparison of electromotive forces.

(8) BEETZ' METHOD. When, as in Experiment 93, a battery current is neutralized by *part* of the current from a more powerful battery, we cannot find the electromotive force of either battery absolutely, unless, as in (3), the whole current from the stronger battery is measured, as well as the resistance which it traverses between the poles of the weaker battery. We may, however, find the relative electromotive forces from formulæ 1 and 2, ¶ 228. Hence if the electromotive force of one battery is known, that of the other may be determined. It may be remarked that by this method we compare the electromotive force of one battery *when at rest* with that of another *when in action.*[1]

(9) CLARK'S POTENTIOMETER. Again, if a current (C) flowing through a resistance R neutralizes one battery (as in Exp. 93), while the *same current* flowing through a resistance r neutralizes another

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[1] By substituting one battery, B, for another, B' (Fig. 254), as the active source in Beetz' Method (Exp. 93) we may compare the two successively with a third electromotive force, B''. This gives us a null method by which we may compare the electromotive forces of two batteries (B and B') *when in action.*

battery (in the same manner), the electromotive forces of these batteries, being CR and Cr respectively, are to each other as R is to r. The proportion between them may therefore be found, independently of any measurement of electrical current. This is the principle of Clark's Potentiometer (Exp. 98), and is undoubtedly the best method of comparing the electromotive forces of two constant batteries *when not in action.*

(10) USE OF CONDENSERS. The relative strength of two batteries may be found by charging a condenser (see ¶ 257) first by one battery, then by the other. The quantity of electricity stored in the condenser is found to be proportional to the electromotive forces in question. It is estimated by discharging the condenser through a ballistic galvanometer, and observing, as in Experiments 76 and 77, the throw of the needle.

(11.) USE OF ELECTROMETERS. The electromotive force of a battery may be determined by connecting the poles with an electrometer (¶ 270); but in order to interpret the indications of the instrument, it must first be calibrated by a series of electromotive forces of known strength. The chief advantage of the use of an electrometer over that of a volt-meter is in the case of inconstant electromotive forces, especially those which disappear as soon as a current begins. The use of a condenser has the same advantage, and is frequently preferable on account of the liability of electrometers to be out of order. Neither instrument is suitable for an elementary class of students.

(12) Use of an Electric Spark. Electromotive forces may be estimated roughly by the distance which an electric spark can be made to jump (see Table 36). This method is particularly suited for experiments with a Ruhmkorff coil, or other instrument in which large differences of potential exist for an instant only.

EXPERIMENT XCIV.

WIEDEMANN'S METHOD.

¶ 231. **Determination of Electromotive Forces by Wiedemann's Method.** — (1) Two Daniell cells, *A* and *B*, one of which (*A*) has been used in Ohm's method (Exp. 92), are to be connected in series with a tangent galvanometer (*C*, Fig. 256, 1). The connections are to be such that the cells act together. The deflection of the galvanometer is to be observed. (2) Then the connections of *B* are to be reversed (Fig. 256, 2), and the deflection again noted. (3) The galvanometer connections are then to be interchanged, and the deflection observed (Fig. 256, 3). (4) Finally the connections of *B* are to be interchanged, so that the two cells may act together as at first (Fig. 256, 4), and the deflection of the galvanometer determined.

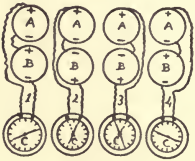

Fig. 256.

Let E be the electromotive force of the stronger cell, and e that of the weaker cell; let A be the average deflection caused by the joint action of the two cells, and C the corresponding current; let a be the average deflection, and c the current produced by the two cells when in opposition; then by formula 7, ¶ 199—

$$C = I \tan A, \tag{1}$$

$$c = I \tan a. \tag{2}$$

Now by Ohm's law (§ 138), as has been explained in ¶ 230, 6, we have—

$$\frac{E + e}{E - e} = \frac{C}{c}, \tag{3}$$

or

$$Ec + ec = EC - eC, \tag{4}$$

whence

$$eC + ec = EC - Ec, \tag{5}$$

or

$$e\,(C + c) = E\,(C - c); \tag{6}$$

from which we find—

$$e = E\,\frac{C - c}{C + c}. \tag{7}$$

Substituting the values of C and c from (1) and (2) and cancelling the factor I, we have—

$$e = E\,\frac{\tan A - \tan a}{\tan A + \tan a}, \tag{8}$$

or

$$E = e\,\frac{\tan A + \tan a}{\tan A - \tan a}. \tag{9}$$

It should be noted that if the reversal of the cell B does not affect the direction of the current,—that is, if the deflections in Fig. 256, 2 and 3, are in the same direction as in 1 and 4 respectively, — the electromotive force of the cell *B*, being less than that of *A*, is to be calculated by formula 8; but if the reversal of *B* causes a reversal of the current, the electromotive force of *B* is greater than that of *A*, and is hence to be calculated by formula 9. The electromotive force of *A*, already computed, may be found from the results of Ohm's method by the formulæ of ¶ 230, 2. The electromotive force of the two cells combined is now to be calculated by adding E and e together.

II. The experiment is to be repeated with the battery composed of the two cells just employed and a Bunsen cell. The cells are first to be set up in series with the Bunsen cell and the galvanometer, then both of the Daniell cells are to be reversed.

The deflections are to be observed and the electromotive force of the Bunsen cell is to be calculated.

EXPERIMENT XCV.

THE THERMO-ELECTRIC JUNCTION.

¶ 232. **Determination of the Electromotive Force of a Thermo-electric Junction** — An iron wire (*ab*, Fig. 257) and a German-silver wire (*ac*), insulated by surrounding them with India-rubber tubes, are soldered together at *a*; and the junction (*a*) is enclosed in a steam heater. The other ends, *b* and *c*, are soldered to insulated copper wires, *bd* and *ce*. The junctions *b* and *c* are placed in a beaker and covered with melting ice. A thermo-element is thus formed with an electromotive force of about 3 thousandths of a volt. The object of this experiment is to measure the electromotive force in question.

I. The terminals of the thermo-element (*d* and *e*) are to be connected with two pole-cups of a differential galvanometer (*dg*) so that the current from the thermo-element circulates in one half of the coil of the galvanometer.

The other half of the galvanometer is to be connected through a rheostat (*hi*) with the poles (*j* and *k*) of a voltaic cell of known electromotive force (¶ 230, 2). There should be at first, let us say, 1000 ohms' resistance in the rheostat. The connections are to be made so that the current from the Daniell cell may produce upon the needle an effect opposite to that due to the thermo-element. The resistance of the rheostat is now to be increased or diminished until the two currents exactly neutralize each other. The rheostat resistance (R_1) is then noted.

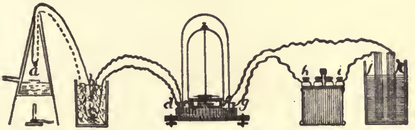

Fig. 257.

An additional resistance (*r*) of known amount, about equal to that of the galvanometer (see Exp. 89), is now to be introduced between *b* and *d*, or between c and e, and the resistance of the rheostat (hi) again adjusted so as to produce equilibrium. The new value of the resistance (R_2) is also to be noted.

II. If a differential galvanometer cannot be obtained, the thermo-electric junction is first to be connected with the galvanometer, and the deflection (D) noted; then the resistance (r) is to be introduced, and the deflection (d) again noted. The Daniell cell is then to be connected with the galvanometer through a resistance (R_1), such that the deflection of the needle is the same as D. Then the rheostat resistance is increased to a value R_2 which produces a deflection equal to d. The results of I. and II. are to be reduced by formula (10), ¶ 233.

¶ 233. **Calculation of the Electromotive Force of a Thermo-electric Junction.** — If in the thermo-electric circuit (*abdeca*, Fig. 257), e is the electromotive force, and b the electrical resistance of the thermo-element, g the resistance of the galvanometer, or that part of it which is included in the circuit in question, c_1 the current in the first part of the experiment, c_2 the current in the second part of the experiment, and r the resistance added; if, furthermore, in the voltaic circuit (*fghijkf*, Fig. 257), E is the electromotive force, B the battery resistance, G the galvanometer resistance, R_1 and R_2 the two rheostat resistances, and C_1 and C_2 the corresponding currents, we have (§ 138), since the currents c_1 and C_1 are equal, —

$$c_1 = \frac{e}{b+g} = C_1 = \frac{E}{B+G+R_1}; \quad (1)$$

and since the currents c_2 and C_2 are equal —

$$c_2 = \frac{e}{b+g+r} = C_2 = \frac{E}{B+G+R_2}. \quad (2)$$

From (1) and from (2) we find —

$$e = E\frac{b+g}{B+G+R_1}, \quad (3)$$

and

$$e = E\frac{b+g+r}{B+G+R_2}. \quad (4)$$

By either of these formulæ (3 or 4) we may calculate the value of e from the observed values of r, R_1, and R_2, if b, g, B, G, and E, are known (Exps. 87–92). The student should bear in mind that the resistance of each part of the galvanometer in this experiment is about twice that of the two parts in multiple arc (§ 140), and half that of the two parts in series. A result independent of the battery and galvanometer resistances may be obtained by combining the observations obtained in the first and second parts of the experiment. Dividing (2) by (1) we have —

$$\frac{b+g}{b+g+r} = \frac{B+G+R_1}{B+G+R_2}, \quad (5)$$

whence

$$\begin{aligned}(b+g)B+(b+g)G+(b+g)R_2 \\ = (b+g)B+(b+g)G+(b+g)R_1 \\ +r(B+G+R_1), \quad (6)\end{aligned}$$

that is, —

$$(b+g)R_2-(b+g)R_1 = r(B+G+R_1), \quad (7)$$

or

$$(b+g)(R_2-R_1) = r(B+G+R_1); \quad (8)$$

from which we find —

$$b+g=\frac{r\,(B+G+R_1)}{R_2-R_1}. \qquad (9)$$

Substituting this value in (3) and cancelling $(B+G+R_1)$, we have finally —

$$e=E\frac{r}{R_2-R_1}. \qquad (10)$$

EXPERIMENT XCVI.

THE VOLT-METER, I.

¶ 234. **Calibration of a Volt-Meter.** — The name volt-meter is given to any instrument capable of indicating directly the value of an electromotive force in volts. One of the forms ordinarily employed (Fig. 258) is similar in external appearance to the ammeter shown in Fig. 231, ¶ 210. There is, however, an essential distinction between these instruments. In the ammeter, the coil a is made so as to have the smallest possible electrical resistance, in order that this resistance may be neglected. In the volt-meter, the finest possible wire is employed in this coil, so that the current which flows through it may be neglected. The simplest way to calibrate a volt-meter is to connect it with a battery containing different numbers of voltaic cells in series (see Fig. 220, ¶ 196). Having found the electromotive force of each cell (see

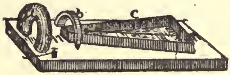

Fig 258.

¶ 230), we may calculate that of the whole battery by adding these electromotive forces together. The difference between this calculated value and the observed reading of the volt-meter gives the correction of the volt-meter for the reading in question. A delicate galvanometer (*G*, Fig. 259) connected in series with a rheostat (*R*) is a convenient substitute for a volt-meter in the measurements relating to the electromotive force of batteries. The resistance in the galvanometer circuit should be so great that we may entirely neglect the current which flows through the instrument in comparison with the other currents used in this experiment. To test such a combination, it is to be connected with a battery of known electromotive force, as for instance, the Daniell cell employed in Experiment 92. If a common astatic galvanometer is employed (Fig. 207, ¶ 188), the resistance of the rheostat should be such as to give a deflection of about 45°. This resistance should be noted, and should remain unchanged through all the experiments with the instrument of which it now constitutes an essential part.

An ordinary astatic galvanometer does not obey the law of tangents (¶ 195) closely enough even for rough determinations. It is necessary, accordingly, to test the reading of the instrument with a series of electromotive forces bearing known ratios to one another.

A simple device by which this object may be attained consists of a uniform straight wire, traversed by a current from a constant battery. The "bridge-wire" of the Wheatstone's apparatus (*hj*, Fig. 259) may be employed. A battery (*B*) of two Bunsen cells in series will probably be required to give the necessary current. The poles should be connected with the ends of the wire by means of screw cups (*b* and *f*) provided for that purpose.

Contact is now to be made between this wire and the terminals of the volt-meter (*GR*) at points 10 *cm.* apart. This may be done by the aid of two sliders, similar to the one used in Experiment 87. Pressure must be exerted upon the sliders to insure a good electrical contact (¶ 193, 11). The deflection of the galvanometer is to be noted. The experiment is to be repeated with contact at two other points the same distance apart, but in a different part of the wire.[1]

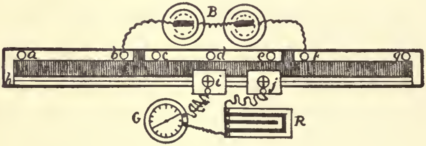

Fig. 259.

The sliders are now to be interchanged and the deflections determined as before.

The direction of each deflection, whether between north and east or between north and west should be noted.

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[1] A record of the reading of each slider corresponding to a given deflection should be preserved, since it may be useful in comparing the resistances of different parts of the wire.

The experiment is now to be repeated with contacts at two points 20 *cm.* apart, then 30 *cm.*, 40 *cm.*, &c., up to 80 or 100 *cm.* (the length of the wire). The observations should be repeated in the inverse order to eliminate variations in the strength of the battery.

The average deflections, corresponding respectively to 10, 20, . . . 80, or 100 *cm.*, are now to be calculated, and the results are to be plotted on co-ordinate paper as is Fig. 260. The distance between the sliders is here represented by a scale at the top of the figure, and the deflections by a scale at the left. The deflection produced by the Daniell cell is also to be plotted, and the number of centimetres corresponding to this deflection found (see § 59). If the electromotive force of the Daniell cell is E volts (¶ 230), and if D is the distance between the sliders which produces an equal current, the distance d corresponding to 1 volt is —

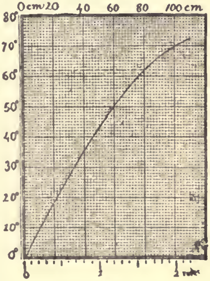

FIG. 260.

$$d = \frac{D}{E}.$$

This distance is to be indicated on the diagram and is to be divided into tenths or smaller parts. The division may be extended across the base of the figure. The theory and uses to be made of the diagram will be explained in the next experiment.

EXPERIMENT XCVII.

THE VOLT-METER, II.

¶ 235. **Determination of Electromotive Forces by means of a Volt-meter.** — A volt-meter, calibrated as in ¶ 234, is to be connected with various cells or batteries, one at a time. The deflection caused by each is to be noted. The electromotive force of each is then to be found (see § 59) by means of the curve already plotted (Fig. 260, ¶ 234). A point a is first located in the scale of degrees corresponding to the deflection in question. Then a point b is found on the curve at the right of a, and below b a point c is found in the scale of *electromotive force* into which the base of the figure has been divided.

The student is to determine rapidly in this way the electromotive forces of all the cells which he has employed.

The principle upon which this method depends is that the difference of potential between two points on a wire of *uniform resistance* is proportional to the distance between those points represented by the scale at the top of Fig. 260. For if R is the resistance of 1 *cm.* of the wire, the resistance of d centimetres will be Rd. Hence from the general formula of § 139 —

$$e = cr = cRd, \qquad (1)$$

$$\frac{e'}{e''} = \frac{crd'}{cRd''} = \frac{d'}{d''}. \qquad (2)$$

If the scale at the bottom of Fig. 260 is constructed so as to give one electromotive force correctly, all electromotive forces should be correctly represented.

EXPERIMENT XCVIII.

CLARK'S POTENTIOMETER.

¶ 236. **Comparison of Electromotive Forces by means of Clark's Potentiometer.** — The positive or carbon pole of a battery (*B*, Fig. 261), consisting of two

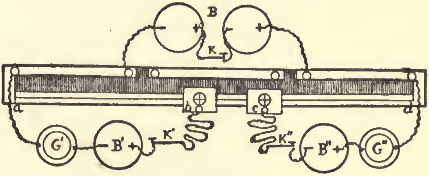

FIG. 261.

Bunsen cells in series, is to be connected with one end, *d*, of a Wheatstone's Bridge wire. The negative or zinc pole is to be connected with the other end (*a*) of the wire. A key, *K*, is to be included in the circuit. The negative (or zinc) pole of a Daniell cell (*B′*) is to be connected with *a*. The positive (or copper) pole is to be joined through a key, *K′*, and a delicate galvanometer, *G′*, to a slider (*b*), by which an electrical connection may be made at any point of the wire. The positive or carbon pole of a Leclanché cell is to be connected similarly with *d*, while the negative (or zinc) pole is to be connected through a key, K'', and a galvanometer, G'', with a second slider at c.

The key K' is first pressed for an instant, and the direction of the deflection noted. Then K and K' are both pressed, the connection being completed first in K then in K'.

If the deflection is in the same direction as before, the distance ab is to be increased; if it is in the opposite direction the distance is to be diminished. The experiment is now repeated until a point b is found such that in pressing both K and K', no deflection is observed. In this case the point b has the same potential as the positive pole of the battery B'.

In the same way a second slider is to be placed at a point c, where the potential is the same as that of the negative pole of the Leclanché cell.

The key K being now closed, the keys K' and K'' are to be pressed simultaneously. If the adjustments have been accurately made, neither galvanometer will be deflected. If this is not the case, the adjustments must be repeated.

By the principle explained in ¶ 235, if the wire ad is of uniform resistance, so that the resistances of ab and cd are proportional to their lengths, the difference of potential between a and b must be to that between c and d as ab is to cd. We have, therefore, —

$$\frac{E''}{E'} = \frac{cd}{ab}, \text{ or } E'' = E'\,\frac{cd}{ab},$$

where E' and E'' represent the electromotive forces, respectively, of the batteries B' and B''. By this formula, knowing the electromotive force of the Daniell cell (¶ 230), we may calculate that of the Leclanché cell. In repeating the experiment, the places of the Daniell and Leclanché elements should be interchanged. If the two sliders should interfere with each other, either 1 or 3 Bunsen cells should be used (in *B*) instead of 2. The experiment may also be repeated with other batteries. Clark's Potentiometer is especially adapted to the determination of the electromotive forces of *inconstant* elements.

EXPERIMENT XCIX.

POGGENDORFF'S METHOD.

¶ 237. **Determination of Electromotive Forces by Poggendorff's Absolute Method.** — The zinc pole *d* (Fig. 262) of a Bunsen battery is to be connected with one terminal (*c*) of the resistance-coil used in the Method of Heating (Exp. 85.) The zinc pole (*a*) of a Daniell cell is to be connected with the same terminal through a delicate galvanometer, *b*. The copper pole (*h*) of the Daniell cell is to be connected with the terminal (*i*) of the rheostat, and the carbon pole (*k*) of the Bunsen cell is to be connected through a tangent galvanometer (*glm*) with the same terminal (*i*).

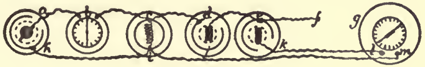

Fig. 262.

A portion (*de*) of a German-silver wire (*def*) having in all a resistance about equal to that of the resistance-coil (*ci*), let us say 1 ohm, is to be included in the circuit of the Bunsen battery.

The wire *def* is to be disconnected for a moment, and the direction of the galvanometer deflection noted. Then the extreme end (*f*) of the wire (*def*) is to be bound in the clamp *e*. If the deflection is in the same direction as before, a longer wire must be employed, and if the two Bunsen cells are still unable to reverse the Daniell cell,[1] other cells must be added to the first, either in series or in multiple arc (§ 140).

We will suppose that a battery (*de*) and a wire (*def*) have been found such that when the wire is clamped at *f*, the current in the Daniell cell is reversed; but when clamped at *d*, the current flows in its natural direction.

The wire (*def*) is next to be clamped at a point (*e*), found by trial, so that the current in the Daniell circuit may be reduced to zero. The galvanometer (*b*) will then show no deflection.

In practice, we clamp the wire at a point (*e*) so that the Daniell cell is barely reversed, and wait for a condition of equilibrium to come about through the gradual weakening of the Bunsen cell. At the moment when the astatic galvanometer (*b*) points to 0° the reading of the tangent galvanometer (*g*) is to be taken.

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[1] The student may be reminded that unless similar poles meet at *c* and at *i*, it will be impossible in any case to produce a reversal of the current.

The experiment is to be repeated with the connections of the galvanometers reversed one at a time, as in Experiment 79.

If a is the mean angle of deflection of the tangent galvanometer and I its reduction factor, the current C is (see ¶ 199, 7) —

$$C = I \tan a \text{ ampères.} \qquad (1)$$

If R is the resistance of the coil (ci) in ohms (Exp. 85) we have a difference of potential (e) between its terminals c and d (see § 139) equal to —

$$e = CR = RI \tan a \text{ volts.} \qquad (2)$$

This is equal to the electromotive force of the *Daniell cell* (see ¶ 130, 3).

For a simplified diagram of Poggendorff's Method, see Fig. 254, 1, ¶ 226. The only change to be made in this diagram is the introduction of a tangent galvanometer in the upper circuit (abc).

EXPERIMENT C.

ELECTRICAL EFFICIENCY.

¶ 238. **Determination of the Efficiency of an Electric Motor.** — A small electric motor, such for instance as is represented in Fig. 263, is to be connected through an ammeter (Fig. 231, ¶ 210) or through a tangent galvanometer (A, Fig. 264), with a voltaic battery (BB) containing at least twice as many cells as are required to keep the motor (M) in motion. Thus if the motor can be started with 2, but not with 1 Bunsen cell, a battery of 4 Bunsen cells should be employed. The poles of the battery are to be connected through a volt-meter or its equivalent (see Exp. 96) consisting of an astatic galvanometer (G) and a rheostat (R). The work done by the motor (M) is to be determined as in Experiment 70, by observing the readings of a pair of spring balances (SS') connected by a cord passing round the pulley of the motor. Ordinary letter-balances will probably answer for this experiment. The tension of the cord should be such as to reduce the speed of the motor

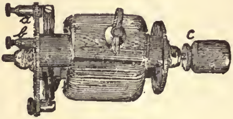

Fig. 263.

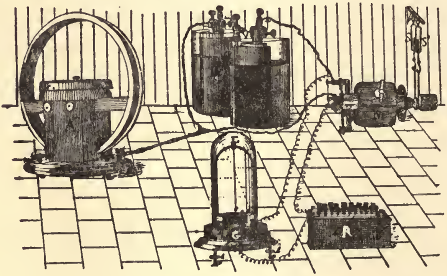

Fig. 264.

to about one half its maximum; but different experiments should be made with different tensions. The number of revolutions made by the wheel of the motor in a given length of time may be determined by an instrument called a "revolution counter" especially devised for this purpose. This consists of a shaft *ab* (Fig. 265) which can be easily connected with the axle of the motor, and a toothed wheel (*c*) with teeth fitting into a thread cut on the shaft at *b*, The revolutions of the shaft are indicated on a dial (*d*) by a pointer (*e*) attached to a wheel (*c*). The circumference of the pulley is to be measured.

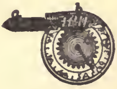
FIG. 265.

Instead of a revolution counter, we may make a band of thread 60 *cm.* long, passing from the pulley of the motor over a second pulley-wheel. Every time that the knot in this band passes a given point shows that the pulley-wheel has advanced 60 *cm.* The velocity of the circumference of the pulley-wheel can be found by this method by counting the number of times that the knot passes a given point in 1 minute. If the band is just 60 *cm.* long, this number represents the velocity in *cm. per sec.* without any reduction.

The power in ergs per sec. utilized by the motor is to be calculated from these data as in ¶ 174, 1, and reduced to watts (§ 15) by dividing by 10,000,000; that is, by pointing off 7 places of decimals. The power in watts spent upon the motor is found by multiplying together the current in ampères indicated by the ammeter (or its equivalent) and the electromotive force in volts indicated by the volt-meter, or its equivalent (see § 137).

The efficiency of the motor is to be found by dividing the power utilized by the power spent (see ¶ 174, 3).

II. Instead of an electric motor, we may employ a small dynamo-machine, driven by a water-motor. The work spent by the water is to be calculated as in Experiment 69. The work utilized is to be found as above by multiplying together the current in ampères and the electromotive force in volts. The former is to be measured by an ammeter in the main circuit of the dynamo-machine ; the latter by a volt-meter connected with the poles of the dynamo-machine. The experiment should be repeated with greater or less resistance interposed in the main circuit.

The student can hardly fail to notice the similarity of the method by which we calculate the work of an electrical current to that used in the case of a current of water (§ 118). The same general method is employed in all measurements of electrical efficiency.

EXPERIMENTS FOR ADVANCED STUDENTS.

The principal methods by which physical quantities are measured have been considered in the course of the 100 experiments which have been described. Various modifications of these methods have already been alluded to. On account, however, of either the practical or the theoretical difficulties involved, and the expense of the necessary apparatus, measurements of certain physical quantities have been hitherto entirely omitted. This course would, however, be incomplete without an outline, at least, of the methods by which some of these quantities may be determined. Most of the experiments about to be mentioned are suitable only for advanced students. For this reason it has been been thought unnecessary to describe them in detail, or to include in the text proofs of the formulæ involved, except when these proofs are necessary to an understanding of the methods employed. The Proofs of other formulæ will be considered separately in Parts III. and IV.

¶ 239. **The Piezometer.** — To measure the compressibility of a liquid, we place it in a glass bulb (*C*, Fig. 266) with a narrow neck or stem (*D*) containing a small mercury index. The bulb is to be placed in a stout glass cylinder filled with water. A consider-

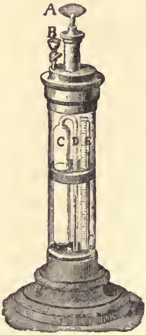

FIG. 266.

able hydrostatic pressure is then generated by means of the thumb-screw, *A*, and measured by a small air manometer, *E* (see ¶ 77). The contraction of the liquid in the stem is observed. Since the bulb is at the same pressure inside and out, there is no tendency to stretch or to crush it. An allowance must, however, be made for the compression of the sides of the bulb. It can be shown geometrically that the capacity of a bulb decreases, when thus subjected to a uniform pressure, in the same proportion as the volume of a solid would decrease under the same circumstances. The ratio of the pressure in dynes per square centimetre to the decrease in volume of 1 cubic centimetre is called the "Coefficient of Resilience of Volume."[1] It is usually calculated from "Young's Modulus" (Y), determined as in Experiment 65, or as in ¶ 248, I., and from the "Simple Rigidity" (S) of a solid. The simple rigidity may be found from the coefficient of torsion, T, (*i. e.*, the couple necessary to twist a wire 1°, see Exp. 64), and from the length, l, and radius, r, of the wire, by the formula —

$$S = \frac{360\ Tl}{\pi^2\ r^4}.$$

It may also be found as in ¶ 248, II. Denoting by M the "coefficient of resilience of the solid," or

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[1] Everett, Units and Physical Constants, Arts. 63–65.

"modulus of volume elasticity," as it is sometimes called, we find —

$$M = \frac{SY}{9S - 3Y}.$$

A mean value of M for glass may be taken as 400,000,000,000 dynes per square centimetre. The quantities S, M, and Y are (in the case of glass and many other substances) related to each other in about the proportion of the numbers 6, 10, and 15 respectively.

If C is the capacity in *cu. cm.* of the bulb (Exp. 11), and P the pressure to which it is subjected, measured in *dynes per sq. cm.*, the contraction of the interior volume of the bulb (V) in *cu. cm.* is —

$$V = \frac{CP}{M}.$$

If V' is the apparent contraction in *cu. cm.* of the liquid, its real contraction is $V + V'$, and the Coefficient of Resilience of volume (M') of the liquid is —

$$M' = \frac{PC}{V + V'}.$$

By making the bulb in two parts, a solid may be introduced into it and surrounded with liquid. The Coefficient of Resilience of the solid may be deduced from its effect on the apparent contraction of the liquid in question.

¶ 240. **Use of a Weight Thermometer.** — If a bulb similar to that employed in ¶ 239, be filled with mercury at an observed temperature t_1, then warmed to the temperature t_2, a certain quantity of mercury will be driven out of it. Let the weight of this mercury be w, and let the whole original weight of the mercury be W_1, both weights being reduced to vacuo (§ 67), then the weight, W_2, remaining in the bulb is $W_1 - w$. If v_1 and v_2 are the specific volumes of mercury at the temperatures t_1 and t_2 (see Table 23, A and B), then the capacities of the bulb (c_1 and c_2) at these temperatures must be —

$$c_1 = W_1 v_1 \text{ and } c_2 = W_2 v_2.$$

It may be shown by geometry that when a vessel is expanded uniformly by heat, its capacity is increased in the same proportion as the volume of a solid would increase under the same circumstances. The cubical expansion, e, of glass is accordingly (see ¶ 63) —

$$e = \frac{c_2 - c_1}{c_1 \,(t_2 - t_1)};$$

hence the linear coefficient, ϵ, is (see § 83) —

$$\epsilon = \tfrac{1}{3}\,\frac{c_2 - c_1}{c_1 \,(t_2 - t_1)}.$$

This is considered to be one of the most accurate methods of obtaining the coefficient of expansion of various kinds of glass.

By collecting and weighing the mercury which is driven out of a bulb or *weight thermometer*, we may estimate the relative rise of temperature in different cases. The instrument is useful in determining precisely the maximum rise of temperature within an enclosure which has to be kept closed at the time when the temperature is taken.

The weight thermometer has also been employed to measure the cubical expansion of solids *enclosed in the bulb.* If c_1 is the capacity of the bulb at the temperature t_1, and if W_1 is the weight of mercury required to fill the space between the solid and the bulb, the volume of the solid V_1 is evidently $c_1 — W_1\ v_1$. If when heated to the temperature t_2, at which the capacity of the bulb is c_2, w grams of mercury are driven out, so that W_2 (or $W_1 — w$) grams remain, then the volume V_2 of the solid is $c_2 — W_2\ v_2$; hence we may find the cubical coefficient of expansion (e) by substituting these values of V_1 and V_2 in the ordinary formula (see ¶ 63) —

$$e = \frac{V_2 — V_1}{V_1\ (t_2 — t_1)}.$$

¶ 241. **Conduction of Heat.** — (I.) The conductivity of various insulating materials may be found approximately by filling the space between the inner and outer cups of a calorimeter (¶ 85) with these materials, and finding the rate at which heat is lost. If A is the mean area of the surfaces between which conduction takes place, L the distance between them, t the difference of temperature, and T the time in which Q units of heat pass from one surface to the other, the specific conductivity (c) of the material is —

$$c = \frac{QL}{tTA}.$$

II. A metallic rod (AD, Fig. 267) is surrounded, one end by steam, the other by melting ice. The central portion is covered with insulating material. Two thermometers, *B* and *C*, are inserted in holes in the rod, partly filled with mercury. If L is the length of the rod between *B* and *C*, *A* the area of its cross-section, *t* the difference of temperature between the points (*B* and *C*), and *w* the weight of ice melted in the time *T*, after a steady flow of heat has been established, *less the quantity melted in the same time when the rod is replaced by insulating material*, then since the latent heat of liquefaction of water is 79, the specific conductivity (*c*) of the rod is given by the formula —

$$c = \frac{79\, wL}{tTA}.$$

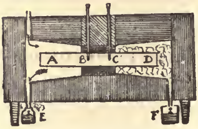

FIG. 267.

The specific conductivity of a given material represents the quantity of heat which would flow in one second from one side of a unit cube made of that material to the opposite side of the cube when the difference of temperature between the two sides is 1°.

The results of this experiment will be slightly modified by the manner in which heat flows through the insulating material which surrounds it. To avoid errors from this source, the distance between the thermometers should be as small as, or smaller than the diameter of the rod. This method should be applied only to metals or to substances which are good conductors of heat.

¶ 242. **Latitude.** — The latitude of a place is usually determined by an observation of the "altitude" of the sun at "apparent noon;" that is, the time when it attains its greatest "altitude," or angular distance from the horizon. The true altitude (a) of the sun is defined as the angle which a line drawn from the centre of the earth to the centre of the sun makes with a plane passing through the centre of the earth and parallel to the horizon of the place in question. The declination (d) of the sun is defined as the angle which the same line makes with the earth's equator. The sun's declination (see Tables 44) may be found in nautical almanacs calculated in advance for every day of each year. The difference between local and Greenwich time, and the hourly change in declination must generally be allowed for. The latitude (l) of a place is by definition equal to the complement of the angle between the horizontal and equatorial planes. We have, accordingly, —

$$l = 90° - a \pm d. \qquad \text{I.}$$

If the sun is (as in summer) above the equator, the sign of d is to be taken as positive; if the sun is below the equator, d is to be called negative.

I. In nautical observations, the apparent altitude of the sun is determined by means of a sextant (see Exp. 44). The lower "limb" (or edge) of the sun is made to coincide with the sea-horizon. The observed altitude (A) must be corrected as follows: —

(1) FOR SEMI-DIAMETER. The apparent semi-diameter (s) of the sun (not far from 16′), given exactly in the nautical almanac for every day in the

year (see also Tables 44), is to be *added* to the observed altitude of the lower limb of the sun, since the altitude of the sun's centre is wanted.

(2) DIP OF THE SEA-HORIZON. A line drawn from the eye of the observer to the sea-horizon makes a certain angle with a true horizontal plane. This is called the "dip of the sea horizon." It may be calculated by the formula —

$$h = \surd\, m \times 1\tfrac{3}{4}' \text{ (nearly)},$$

where m is the height in metres of the eye above the sea-level. The dip (h) must be subtracted from the observed altitude.

(3) FOR REFRACTION. Atmospheric refraction tends to make heavenly bodies appear higher than they really are. The correction (r) is accordingly to be subtracted from the observed altitude. It is given by the equation —

$$r = \mathit{cotan}\ A \times 1' \text{ (nearly)}.$$

(4) FOR PARALLAX. The apparent altitude of a body as seen from the earth's surface is obviously less than if it could be observed at the earth's centre. In the case of the stars, on account of their enormous distance, the difference is imperceptible. The correction for parallax (p) is given in general by the equation —

$$p = P \mathit{cos}\ A \text{ (nearly)},$$

where P is the "horizontal parallax" of the body in question; that is, its correction for parallax when seen on the horizon. In observations of the sun with an ordinary sextant, since P is less than 9″, all corrections for parallax may usually be neglected. It is only in the case of the moon, where P is in the neighborhood of 1°, that the correction for parallax becomes important.

The true altitude (a) of a heavenly body is found in general from the observed altitude (A) by applying the corrections for semi-diameter (s), dip of the horizon (h), refraction (r), and parallax (p) as follows:

$$a = A + s - h - r + p. \qquad \text{II.}$$

II. Observations of latitude taken on land are usually made with an "artificial horizon." This may consist of a plate-glass mirror (made horizontal by two spirit-levels and levelling-screws) or simply a dish of mercury (B, Fig. 268) The lower limb of the sun is made to coincide with its own reflection in the horizontal surface. The observed angle (D) between the direct and reflected rays reaching the sextant (A, Fig. 268) is measured, and *halved*, to find the apparent altitude of the sun. The result is corrected as above for semi-diameter, refraction, and (if sufficiently accurate) for parallax.[1] We have —

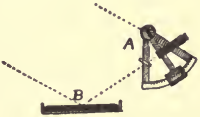

FIG. 268.

$$a = \tfrac{1}{2}D + s - r + p. \qquad \text{III.}$$

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[1] The correction for "dip" is obviously to be omitted in the case of an "artificial horizon," since the plane of the reflecting surface should be perfectly horizontal.

The latitude is finally calculated by formula I. above.

¶ 243. **Longitude.** — The longitude of a place may be determined by a sextant observation of the altitude of the sun (see ¶ 242) an hour or two after sunrise or before sunset. For the reduction of the results, in general, the student is referred to works on navigation. A simple (though not very accurate) method of finding the longitude of a place is to measure the altitude of the sun at an observed time t' (about an hour before noon), then to determine exactly the time t'' (about an hour after noon) when the sun descends to the *same altitude*. Obviously the time of "apparent noon," t, (neglecting the change in the sun's declination), is half-way between t' and t'', that is —

$$t = \tfrac{1}{2}(t' + t''), \text{ nearly.} \qquad \text{I.}$$

If e is the "equation of time" (see Tables 44), given in all nautical almanacs, the time (T) of "mean noon" is (by definition) given by the formula —

$$T = t \pm e. \qquad \text{II.}$$

The sign of the quantity e is positive if the sun is fast, but negative if the sun is slow.

It is assumed that the chronometer employed in this experiment has been set so as to indicate correctly the time of a given meridian, as for instance that of Greenwich, from which it is desired to measure longitude. If it does not indicate this time correctly, an allowance must be made for the error of the chronometer. At sea, several chronometers are frequently carried. In certain cases a chronometer may have to be set by a lunar observation. For the reduction of such results (which is exceedingly complicated), the student is referred to works on navigation. On land, the standard time of a given meridian is usually obtainable by means of the electric telegraph.

It may be remarked that the longitude of a place is given by formula II. in hours, minutes and seconds.

¶ 244. **Indices of Refraction.** — I. If A is the angle of a prism (Exp. 45), and D the angle of minimum deviation (Exp. 46) of a ray of light of a given wave-length, the index of refraction (μ) of the material of which the prism is composed is (for light of that wave-length) —

$$\mu = \frac{\sin \frac{1}{2} (A + D)}{\sin \frac{1}{2} A}.$$

Certain "doubly refracting" substances have two indices of refraction instead of one. To determine them we employ a prism cut so as to produce the maximum separation of the two rays into which a single ray of monochromatic light can be decomposed by the given prism angle. The minimum deviation of *each* ray is then measured, and the two indices of refraction are calculated separately by the ordinary formula.

II. If R is a mean radius of curvature of the two surfaces of a double convex lens (Exp. 21), and F its principal focal length (Exps. 41–43), the index of refraction of the material of which the lens is made may be found by the formula —

$$\mu = 1 + \tfrac{1}{2}\frac{R}{F}.$$

If the same lens (*B*, Fig. 269) be enclosed between two flat glass plates (*A* and *C*), and the space be filled with a liquid, with the index of refraction μ', then if F' is the principal focal length of the combination, we have —

$$\mu' = \mu - \tfrac{1}{2}\frac{R}{F'}.$$

If R_1 and R_2 are the two radii of curvature of the two sides of the lens, the mean radius of curvature should strictly be calculated by the formula —

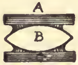

FIG. 269.

$$R = \frac{2\,R_1\,R_2}{R_1 + R_2}.$$

¶ 245. **Polarization.** — The vibrations which constitute ordinary light are, according to modern theories (§§ 92, 93), at right-angles with the direction in which the light is propagated. In a vertical beam of light, for instance, the vibrations are supposed to be confined to a horizontal plane. The vibrations appear in general to be distributed uniformly in every possible direction perpendicular to the path of the ray. Certain substances and certain optical combinations have, however, the property of stopping all the vibrations — or rather all their *components* (§ 105) — except those in a certain direction, as for instance north and south. The light transmitted is then said to be polarized.

In many optical instruments, light passes successively through two such combinations. The first is called the "polarizer" (*e*, Fig. 270), the second is called the "analyzer" (*a*). If the polarizer and analyzer are placed so that the direction of the vibrations transmitted is the same in both cases, the light which has passed through one will also pass freely through the other; but since the polarizer transmits only vibrations in a given direction, if the analyzer is placed so as to stop all vibrations in this direction, a beam of light which has passed through the polarizer will be completely cut off by the analyzer. The position of the analyzer when this occurs is indicated by a pointer attached to it. The reading of the pointer with respect to the graduated circle *b* determines the zero-reading of the instrument.

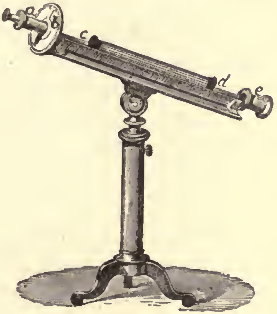

Fig. 270.

Certain substances have the property of changing the direction of the vibrations in a beam of polarized light which they transmit. Thus in passing *upward* through a solution of cane sugar, north and south vibrations are gradually changed into a northeast and southwest direction.[1]

When a substance producing "rotation of the plane of polarization" is placed between the polarizer and the analyzer in its zero-position, the analyzer will no longer cut off all the light transmitted by the polarizer. To produce perfect darkness, the analyzer must obviously be turned through an angle equal to that through which the plane of polarization has revolved. The instrument shown in Fig. 270 affords, accordingly, a means of measuring the rotation of the plane of polarization.

To test the strength of a solution of sugar with this instrument, we pour the solution into a tube *cd* with glass ends, and interpose the tube in the path of the beam *ea* of polarized light. The analyzer is then turned to the right from its zero-position, until the light which it transmits is reduced to a minimum.

---

[1] When light is polarized by reflection, it is said to be polarized in a plane perpendicular to the reflecting surface, and containing both the incident and the reflected rays. According to Fresnel's theory the vibrations in a beam of polarized light take place at right-angles with the "plane of polarization." The action of a solution of sugar upon a beam of polarized light *approaching the eye* is to rotate the plane of polarization (and hence also the direction of the vibration) *with the hands of a watch.* The student should note that this is called a right-handed rotation in optics; but that it is opposite to the motion of an ordinary right-handed screw, which when turned to the right moves *away from* the eye.

Let a be the angle in degrees through which it is turned when *sodium light* is employed, and let d be the depth of the sugar solution, equal to the distance between the glass ends of the tube cd; then experiments show that the strength of the solution (s) in grams per *cu. cm.* is given by the equation (Kohlrausch, § 46), —

$$s = .15 \frac{a}{d} \text{ (nearly).}$$

The rotation varies considerably with lights of different colors (see Table 31 E). For this reason, when ordinary white light is employed perfect darkness can never be attained.

There are various optical effects (besides the darkness produced by an analyzer) which depend upon the plane in which light is polarized. Many of these have been applied to the determination of angles of rotation of the plane of polarization. The method described above has been chosen because of its simplicity.

¶ 246. **Color.** A piece of colored paper (c, Fig. 271) may be mounted in front of a white screen (d) and illuminated by a candle (a) through a piece of ruby glass (b), all other light being cut off. The distances ac and ad must be adjusted so that c and d appear equally bright when viewed from a point near b. The "relative luminosity" of the surface c is then equal to $(ac)^2 \div (ad)^2$ as far as reflected red rays are concerned.

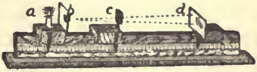

Fig. 271.

A transparent gelatine plate stained with an emerald green mixture of common green and yellow inks is now substituted for the ruby glass (*b*), and the relative luminosity is again determined. Finally, a gelatine plate stained with a violet mixture (Hofmann's violet containing a trace of soluble Prussian blue) is employed.

The three relative luminosities of the surface *c*, obtained as above by means of red, green, and violet rays, completely determine the color of the surface in question (see ¶ 115).

¶ 247. **Velocity of Light.** — The velocity of light was determined by Fitzeau in 1849.[1] A beam of light made intermittent by passing between the teeth of a revolving wheel, was sent to a distant mirror, then reflected back to the eye through the same wheel. When the wheel (which had 720 teeth) made 12.6 revolutions per second, the flashes of light, in traversing a total distance of 17,326 metres, were retarded so as to strike a tooth instead of the space between two teeth; hence the light was cut off. When the speed of the wheel was doubled, so that 18,144 teeth passed a given point in one second, the light reappeared; when trebled it disappeared, &c. It was inferred from this experiment that a beam of light required $\frac{1}{18144}$ of a second to traverse 17,326 metres; whence the velocity of light would be about 18,144 × 17,326 metres per second, or nearly thirty thousand million *cm. per sec.*

---

[1] See Deschanel's Natural Philosophy, § 686; Ganot's Physics, § 507.

Foucault has measured the time required by light to traverse short distances (a few metres only) by the use of a revolving mirror.[1] A beam of light (*AB*, Fig. 272) striking the mirror (*B*) was reflected to a fixed concave mirror (*CC′*) *with its centre of curvature in the axis of the revolving mirror* (*B*), then back on its course to the revolving mirror (*B*), and thence to the eye. The beam strikes the eye only for a very short time during each revolution of the mirror, but on account of the rapidity of rotation a continuous effect is produced. When the speed of rotation reaches several hundred revolutions per second, the mirror turns through a perceptible angle while the light is passing from *B* to *C* or to *C′* and back again.

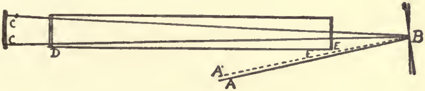

Fig. 272.

Hence the return path *BA′* differs slightly from the original path *AB*.

With a distance *BC* equal to about 4 metres, and with from 600 to 800 revolutions per second, divergences of about 40″ or 50″ were observed. The velocity of light was found to be 29·8 (or nearly 30) thousand million *cm. per sec.*

By passing the beam of light through a tube of water (*DE*, Fig. 272) it was found that the velocity of light in water is about $\frac{3}{4}$ that in air.

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[1] Deschanel's Natural Philosophy, § 687; Ganot's Physics, § 506.

¶ 248. **Velocity of Sound in Wires.** — I. If a wire stretched between two vises be stroked horizontally near one end by a piece of resined cloth, a musical note may result from the *longitudinal vibrations* into which the wire is thrown. The pitch of the note is to be determined by a "pitch pipe" (Fig. 273) or any instrument serving a similar purpose. The number of vibrations corresponding to the note may be found by reference to Table 43. If l is the length of the wire between the vises, and n the number of vibrations per second, the velocity of sound (v) is —

FIG. 273.

$$v = 2nl.$$

II. If a strip of resined cloth be drawn slowly *round* the wire (like a belt round a pulley) a musical note may result from *torsional vibrations* set up in the wire. The velocity of these torsional vibations may be found by the same formula as above. The note due to longitudinal vibrations is usually about a "sixth" (¶ 134) above that due to torsional vibrations. Hence the two velocities of sound are to each other as 5 to 3, nearly.

If d is the density of the wire, Y Young's Modulus of Elasticity (¶ 166) and S the simple rigidity of the wire (¶ 239) v_1 and v_2 the velocities of longitudinal and torsional vibrations, we find —

$$Y = v_1^2 d. \qquad \text{I.}$$

$$S = v_2^2 d. \qquad \text{II.}$$

¶ 249. **Reversible Pendulum.** — A reversible pendulum (Fig. 274) may be made of cast iron,[1] so that although the two knife-edges A and B are at very unequal distances from the centre of gravity (C) the time of oscillation on both knife-edges is nearly the same. The position of C must be found approximately (Exp. 62), and the distances AC and BC measured. The distance AB must be accurately determined (by measuring DE, DA, and BE with a vernier gauge, and subtracting DA and BE from DE). If t' is the time of oscillation on the knife-edge A, and t'' that on B (see Exp. 58), the time t of oscillation of a simple pendulum of the length AB is —

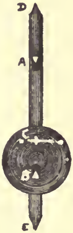

FIG. 274.

$$t = t' + \frac{BC}{AC - BC}(t' - t'').$$

Denoting by l the distance AB, the acceleration of gravity (g) may now be calculated by the ordinary formula —

$$g = \frac{\pi^2 \, l}{t^2}.$$

---

[1] For a half-seconds pendulum, the following dimensions are suggested: extreme length of the shaft (DE), 45 *cm.*, breadth $3\frac{1}{3}$ *cm.*, thickness 1 *cm.*; ends sharpened to an angle of about 70°; triangular knife-edges (steel better than cast iron) 2 *cm.* long, sides 1 *cm.* broad; distance of each knife-edge from nearest extremity, 10 *cm.*; holes 1 × 2 *cm.*; disc 14 *cm.* in diameter, 2 *cm.* thick; centre of disc 24 *cm.* from one knife-edge, 1 *cm.* from the other. This pendulum should weigh about 3 kilograms. The centre of gravity should be about 5 *cm.* from one knife-edge, and 20 *cm.* from the other. In observations of its time of oscillation, the knife-edges may rest upon the upper surface of a short steel rod, 7 *mm.* square, driven horizontally into the wall.

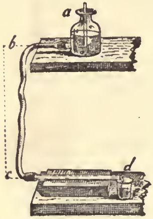

Fig. 275.

¶ 250. **Coefficient of Viscosity.** — A liquid contained in a Mariotte's bottle (a, Fig. 275) is fed through a rubber tube (bc) into a capillary tube (cd), and collected in a small vessel (e). The weight (w) which passes through the tube in a given length of time (t) is found, and the height (h) of the inlet (b) above the orifice (d) is determined. The length (l) of the tube (cd) is measured, and its radius (r) is found (see ¶ 170). Then if d is the density of the liquid (Exp. 14), and g the acceleration of gravity (Exp. 58), the coefficient of viscosity of the liquid is given by the formula, —

$$\eta = \frac{\pi g d^2 h r^4 t}{8\, w l}.$$

This coefficient of viscosity is the force in dynes necessary to maintain a difference of velocity equal to 1 *cm. per sec.* between two opposite faces of a centimetre cube.

The ordinary coefficient of liquid friction (see ¶ 172) depends upon the *square* of the velocity, and has no relation to the coefficient of viscosity.

¶ 251. **Electro-chemical Equivalents.** — If, in Experiment 81, I is the reduction factor of the galvanometer, determined as in Experiment 83, w the weight of copper deposited by the current C in the time t, and a the average angle of deflection, we have for the electro-chemical equivalent (q) of copper —

$$q = \frac{w}{Ct} = \frac{w}{t\, I \tan a}.$$

By the same formula we may find the electro-chemical equivalent of any other substance acted upon by the current C, whether that action be to deposit the substance in question, or to cause it to go into solution. In the case of a gas set free at one of the electrodes of a voltameter (C or D, Fig. 276), we find the weight indirectly from the volumes collected in graduated tubes (A and B), originally filled with the liquid (E) which is decomposed by the current. A battery of two or three Bunsen cells should be used with a gas voltameter.

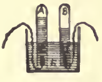

FIG. 276.

If w', w'', w''', &c., are the weights of different substances acted upon by a given current traversing a series of voltameters for a given time, the electro-chemical equivalents q', q'', q''', &c., may be found (if any one is known) from the proportion —

$$w' : q' :: w'' : q'' :: w''' : q''', \&c.$$

¶ 252. **Correction of Rheostats.** — An arrangement of a set of resistances, convenient for the purposes of correction, is represented in Fig. 277. The outer horse-shoe (ad) contains 18 coils capable of being combined so as to give any resistance under 4,000 ohms, within one tenth of an ohm. The inner horse-shoe ($befc$) contains resistances arranged in pairs of 1, 10, 100, and 1000 ohms each. Opposite a and c are two extra blocks. These are permanently

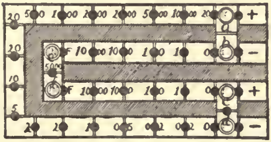

FIG. 277.

connected together, underneath, by a thick copper rod. One of them is joined to the positive pole of a battery. Two blocks opposite *b* and *d* are similarly joined together, and one of them is connected with the negative pole of the battery.

One terminal of the galvanometer is now carried to *e* (or to *f*). The other terminal is to be connected with one of the blocks in the outer line of resistances *between two coils*, or sets of coils, which are to be compared. A pair of resistances about as great as the coils in question is now introduced into the inner horse-shoe. When the battery is connected with *a* and *d*, the rheostat assumes the form of a Wheatstone's Bridge (§ 141). The inner horse-shoe furnishes two of the arms *be* and *cf*. The connections of these arms may be interchanged by breaking the battery connections at *a* and *d*, and making them at *b* and *c*. The arrangement of blocks furnishes in fact a commutator within the box of coils. By the use of this commutator, errors due to inequality in a given pair of resistances may be eliminated (§ 44).

The 1-ohm coil is first to be tested against the smaller coils, together equal to 1 ohm; then joined in series with the smaller coils, and tested against each of the 2-ohm coils; then the 5-ohm coil, the 10-ohm coil, &c., are to be tested each against its equivalent in terms of the coils below it in the line of resistances. If differences are observed, the sensitiveness of the galvanometer to a change of 1 ohm (or 0.1 ohms) in the outer line of resistances must be determined. The differences in question may then be estimated by interpolation (see ¶ 216). The results are to be reduced as in ¶ 217. When the ratios of the different coils in the outer series have been found, that of any pair of coils in the inner horse-shoe may be determined by comparison.

¶ 253. **Resistance of Electrolytes.** — We may substitute in Exp. 87 an alternating current for a common battery current; in this case the galvanometer must be replaced by some instrument like the dynamometer, sensitive to alternating currents. A telephone is sometimes found to give satisfactory results with a rapidly alternating current. Usually a loud note is heard in the telephone; but when the Wheatstone's bridge is in adjustment, the sound either completely ceases or reaches a minimum.

The advantage of using alternating currents is that, in the short time during which they last, the effects of polarization are so small as to be almost inappreciable. The method is especially valuable in the determination of the resistances of batteries and electrolytes. It is not, however, always successful, on account of various causes tending to destroy the minima of sound. To obtain satisfactory results, the resistance to be measured should be not less than 10 or 15 ohms. The electrodes should consist of platinum strips, at least 10 *sq. cm.* in area, and freshly coated with platinum through electrolytic action (Kohlrausch, 6th ed. 72 II.),

¶ 254. **Measurement of Electrical Capacity.** — A "condenser" consists of two sets of thin metallic plates, arranged alternately, as in Fig. 278, so that although the plates are very close together, there is no metallic connection between the two sets. The plates are generally separated by thin layers of glass, mica, or paper dipped in paraffine. The plates of one set are all connected with one binding-post (A); those of the other set with another binding-post (B). A condenser is charged by connecting A and B each with one pole of a battery. It may then be disconnected from the battery, and discharged through a galvanometer by carrying the terminals to A and B. Care must be taken not to touch both terminals at the same time.

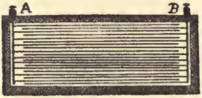

Fig. 278.

The capacity of a condenser is defined as the quantity of electricity which can thus be stored in it by a battery having an electromotive force equal to 1 unit *in absolute measure.* The "farad" is a thousand millionth part of the electro-magnetic unit of capacity. The distance between the plates of a condenser is usually very small in comparison with the area of the separate plates. To calculate the electrical capacity of such a condenser, we measure the thickness (t) and total area (A) of the insulating layers, then if s is the "specific inductive capacity" of the insulating material (¶ 256), the capacity (C) of the condenser is given in electrostatic units by the equation —

$$C = \frac{As}{4\pi t}, \qquad \text{I.}$$

or, since it has been found by experiment that 1 microfarad is equivalent to about 900,000 electrostatic units,[1] the capacity (c) in microfarads may be calculated by the formula —

$$c = \frac{As}{36{,}000{,}000\,\pi t} \text{ microfarads (nearly).} \qquad \text{II.}$$

The specific inductive capacity (s) of the insulating material must in general be found as in ¶ 256; but when the plates of a condenser are separated by air spaces, since the specific inductive capacity of air is taken as 1, the capacity of a condenser may be calculated from direct measurements of the area and thickness of the insulating material.

The capacity of any condenser may be determined by measuring the quantity of electricity stored in it by a battery of known electromotive force. With the aid of clockwork, a condenser is to be charged by a battery and discharged through a galvanometer n times a second; the deflection of the galvanometer being noted. Then if R is the resistance in ohms through which the same battery produces the same deflection (see Exp. 95, II.) we have —

$$c = \frac{1{,}000{,}000}{nR} \text{ microfarads.} \qquad \text{III.}$$

In practice we must employ a very sensitive galvanometer capable of measuring currents at least in millionths of an ampère. The time of oscillation of the needle should be 10 seconds or more, in order that the intermittent discharge through the instrument may produce a sensibly constant effect. An ordinary condenser of 1 microfarad capacity cannot be charged and discharged satisfactorily more than 10 or 100 times per second.[1] To avoid large errors due to this cause, the speed of the mechanism should be reduced until an approximate agreement is obtained between two or more results.

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[1] **Everett, Units and Physical Constants, Arts. 177, 185.**

The experiment may be performed with an ordinary astatic galvanometer, but only by the use of a condenser of great capacity and a battery of high electromotive force.

¶ 255. **Comparison of Condensers.** — The capacities of two condensers may be compared by charging them, successively, by a given battery, then discharging them successively through a ballistic galvanometer (see ¶ 187). The capacities will then be approximately as the chords of the throws (§ 109).

The capacities of two condensers may be compared with great precision by including the condensers in two adjacent arms of a Wheatstone's bridge (see Exp. 87). One pole of the battery must be applied between the two condensers. The resistances in the other two arms of the bridge should be great, and adjusted so that a sudden *reversal* of the battery current causes no sudden deflection of the galvanometer.[2] If C_1 and C_2 are the capacities of the two condensers, R_1 and R_2 the resistances adjacent to them, respectively, we have —

---

[1] Owing to effects of "electrical absorption" and "residual charge," the quantity of electricity stored in or obtained from a condenser depends somewhat upon the time during which connections are made. See Ganot's Physics, § 773. When a condenser is rapidly charged and discharged, these phenomena almost entirely disappear; but the resistance of the various conductors may reduce the quantity of electricity which can flow in and out of the condenser to an indefinitely small amount.

[2] See Glazebrook and Shaw, Practical Physics, §§ 81, 82.

$$C_1 : C_2 :: R_1 : R_2.$$

We have seen (¶ 254) that the capacity of a condenser with air spaces between its plates may be measured. The capacity of such condensers is generally so small that comparisons cannot be made by ordinary methods. By substituting an alternating current for the battery and a telephone for the galvanometer (see ¶ 253) in the combination described above, comparisons of these and even smaller capacities should be possible.

¶ 256. **Specific Inductive Capacity.** — When two condensers are similar in every respect except the nature of the insulating materials used in their construction, their capacities (c and c') are to each other as the "specific inductive capacities" (s and s') of these materials. Since the specific inductive capacity of air may be taken as 1, we have in general, from ¶ 254, I.,—

$$s = \frac{4\pi ct}{A}.$$

The specific inductive capacity of a given insulating material may accordingly be found by constructing a condenser with that material between its plates, measuring the area of and distance between these plates, and determining as in ¶ 254 or as in ¶ 255 the capacity of the condenser.

Winkelmann's method for testing specific inductive capacities consists in the use of three parallel plates, *A*, *B*, and *C* (Figs. 279 and 280), equal in area, and 15 or 20 *cm.* in diameter. *A* and *B* are separated by an air space of the thickness *a*, while *B* and *C* are separated by an air space of the thickness *b*, and by a thickness *c* of the material whose specific inductive capacity is to be determined. The outer plates *A* and *C* are connected either through a telephone (*T*, Fig. 279) with each other, or through a differential telephone (*DT*, Fig. 270), and through a metallic conductor (*G*) with the ground. The central plate (*B*) is joined to one pole of an induction coil, the other pole of which is connected through *G* with the ground. The distances *a* and *b* are then adjusted so that the sound heard in the telephone is reduced to a minimum. The specific inductive capacity (*s*) is then given by the formula —

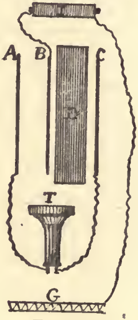

Fig. 279.

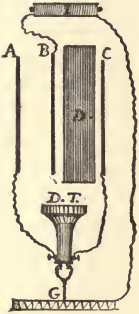

Fig. 280.

$$s = \frac{c}{a - b}.$$

In Winkelmann's method we may consider that the plates *A* and *B* form one condenser, while the plates *B* and *C* form another condenser. When the capacities of these two condensers are equal, a given charge of electricity on *B* must raise *A* and *C* to the same potential; hence *if the effect be simultaneous* no current will flow through the telephone. In practice, most dielectrics cause a slight retardation in the charging of a condenser, so that although the telephone gives a minimum of sound, it never becomes perfectly silent.

¶ 257. **Comparison of Electromotive Forces by means of a Condenser.** — The pole cups of a condenser (*A* and *B*, Fig. 278) are to be connected as in ¶ 254 with the poles of a battery, then disconnected from the battery, and connected with the terminals of a ballistic galvanometer, the throw of which is to be
observed. The experiment is to be repeated w
second battery. If a' and a'' are the throws
E'' the electromotive forces, we have (s
the angles are small, —

$$\frac{E'}{E''} = \frac{chord\ a'}{chord\ a''} = \frac{a'}{}$$

In this experiment it is i
of charging, discharging,
should be exactly the sam

¶ 258. **Electrostatic Syst**
(*b* and *c*, Fig. 281), of equal
ter (*d*) are both to be suspen
point *a*, by fine cotton threads

The threads may be blackened with a lead-pencil to make sure that they will conduct electricity. One pole of a battery (*de*), of several hundred volts, is to be connected with the point (*a*) of suspension; the other pole with the ground.

The balls *b* and *c*, being similarly charged, will now repel each other. A considerable divergence should be observed. The distance (*s*) between the centres of the two balls is to be found by a sextant placed at a fixed distance (see ¶ 124). The electro-

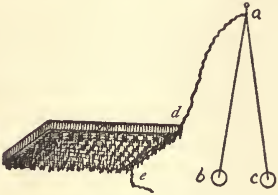

FIG. 281.

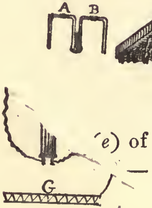

FIG. 279.

´e) of the battery in electrostatic units

$$\sqrt{\frac{2\,wgs^3}{ld^2}}.$$

(*B*) is joined to one po
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The cords *ab* and *ac*
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$s =$

g. 281) will be found con-
ment. It may be constructed

of alternate strips of zinc and copper soldered together in pairs and attached with pitch to the under side of a board so that drops of water or dilute sulphuric acid may be taken up between adjacent pairs (as *A* and *B*).

It has been found by experiment that one unit of electromotive force in the electrostatic system is equal to about 300 volts, or 30 thousand million absolute units in the electromagnetic system. It is an interesting fact that the ratio between the absolute units of the two systems is equal, within the limits of errors of observation, to the velocity of light (see § 93).

INSTRUMENTS OF PRECISION.

The apparatus employed in the course of experiments which has been described is of the simplest possible form. The most accurate results can be obtained only by the use of instruments especially designed for a given purpose. The following sections contain a brief description of the construction and adjustments of certain instruments of precision, which though unsuitable for an elementary class of students, might be advantageously employed by advanced students in place of the ordinary apparatus.

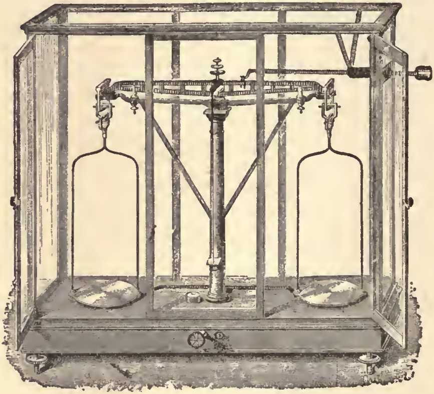

Fig 282.

¶ 259. **Analytical Balances.** — The adjustments of an analytical balance (Fig. 282) and the precautions in using it are essentially the same as those described in Experiment 6. In addition to the mechanism, operated from outside the case, by which in a fine balance *all* weight may be removed from the knife-edges, there is often a pan-arrester, which has to be moved before two weights can be exactly balanced. A preliminary adjustment of the weights should be carried as far as centigrams on an ordinary balance. The weights may then be transferred to the analytical balance, and a finer adjustment made by means of a rider (*e*, Fig. 283) made of platinum wire. The rider can be placed at any point (*e*) of a graduated scale on the balance-beam by means of a hook (*d*) attached to a rod (*ac*) passing through a tube (*b*) in the side of the balance-case. The necessary motion is given to the hook by pushing, pulling, or twisting the rod (*ac*).

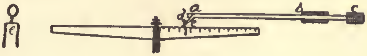

Fig. 283.

The indication of the pointer is always found while it is in oscillation (¶ 20); but since the weights may be adjusted by means of the rider with any degree of precision, the method of interpolation (¶ 20), though generally quicker, need not be employed.

In finding the position of the rider necessary for an exact balance, the same method of approximation should be employed, at first, as in the adjustment of weights; that is, the rider should be placed midway between two distances on the scale, one too great the other too small, until the deflection of the pointer and the sensitiveness of the balance indicate directly where it should be placed. When finally observations of the swings of the pointer show that it would come to rest at its zero-position, the position of the rider is noted.

The accuracy of the rider is tested by weighing a small weight with it. To obtain results accurate to a tenth of a milligram, the set of weights employed (even the best) should be most carefully tested (¶ 25).

The advantage of weighing with a rider is that the final adjustment of two weights may be made with the balance-case closed. The air within the case should always be kept perfectly dry with chloride of calcium (or with concentrated sulphuric acid), which must be renewed from time to time. Neither arm of the balance should be exposed to the heat of a fire or lamp, or to the cold glass of a window. The method of double weighings should if possible be employed. If it is not employed, care must be taken that the pans are equal in weight, and that in the zero-position, the balance-beam is horizontal and the pointer vertical.[1]

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[1] When the greatest accuracy is desired, arrangements must be made to carry on the ordinary processes of weighing from a distance. Thus at the International Bureau of Weights and Measures at St. Cloud, not only the suspension of weights from the balance-beams, but also the interchange of the contents of the scale-pans is accomplished by a series of shafts leading from each instrument nearly to the centre of a large room in which the finest balances are contained. Mechanical contrivances are also employed for the final adjustment of weights *in vacuo*.

¶ 260. **Comparators.** — A simple form of comparator is represented in Fig. 284. It consists of two reading microscopes (*A* and *B*) mounted on supports (*E* and *F*) which slide along a rail (*GH*). The sliding supports may be clamped at any point of the rail by thumb-screws (*C* and *D*). A small scale of tenths of millimetres (*b* and *b′*, Fig. 284) is placed in the tube of each microscope at a distance from the object glass (*c*) equal to twice its focal length. The eye-piece (*a*) is first focussed upon this scale, then raised or lowered until a given object is in focus. Let us suppose that the two microscopes are thus set, one upon each end of a scale. It is obvious that if a standard scale be now substituted any difference between the two will be not only readily detected, but easily measured in tenths of a millimetre and such fractions of a tenth as may be estimated by the eye (§ 26).

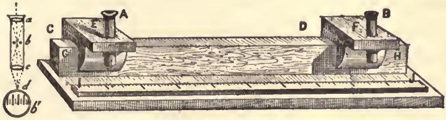

Fig. 284.

Care must be taken to have the *upper* surfaces of the two scales on the same level, so that both scales may be in focus, and to have the microscopes firmly clamped, and not subjected to any strain between observations.

¶ 261. **The Dividing-Engine.** — A dividing-engine (Fig. 285) consists essentially of a micrometer (*c*) with a long screw (DG) fixed in position, so that when the micrometer is turned, a nut (EF) gives a slow motion to a slide (B) to which a reading microscope (A) is usually attached. The length of an object parallel to the screw is determined by the number of turns of the micrometer necessary to make the microscope travel from one end of the object to the other. The microscope is of course provided with cross-hairs, so that it may be set exactly on a given point. The screw is always to be turned in a given direction in measuring a given distance; otherwise an error due to looseness of the screw ("backlash") may be made. The pitch of the screw in different parts is found by measuring with it a standard scale of known length (see ¶ 52). If the nut is long and fits equally well in all parts of the screw, no *great* variations of pitch can occur.

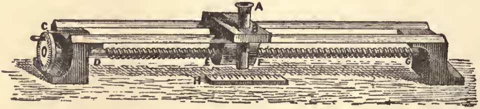

FIG. 285.

The dividing-engine is especially useful in measuring distances between the lines of a scale, or lengths of columns of mercury in the calibration of a tube (see ¶ 71). The results may be more precise than those obtained with any other instrument for the measurement of length.

¶ 262. **The Cathetometer.** — (*κατὰ*, down, *τίθημι*, to place, and *μέτρον*, measure) is an instrument for measuring vertical distances (Fig. 286). It consists of a horizontal telescope or reading microscope (*b*) sliding on a vertical shaft (*ah*), which is capable of rotating about its own axis. Sometimes the shaft is graduated, the carriage to which the telescope is attached being provided with a vernier, so that the height of the telescope may be read. Slow motion may also be given by a micrometer screw (*ef*). The cathetometer may then be used for measuring small vertical distances, just as the dividing-engine (¶ 261) is used for horizontal distances. The micrometer is useful in measuring precisely, for instance, the distance through which a wire is stretched (Exp. 65). For ordinary purposes, neither the micrometer nor the vernier is required. The shaft is first adjusted by the eye so as to be as nearly perpendicular as possible, by means of the levelling-screws (*h*, *i*, and *l*) at the base of the instrument, then the telescope is made horizontal according to a spirit-level (*c*) with which it is provided. Then the shaft is rotated about its axis. If the axis is not vertical, the bubble in the spirit-level will tend to move in a given direction. The

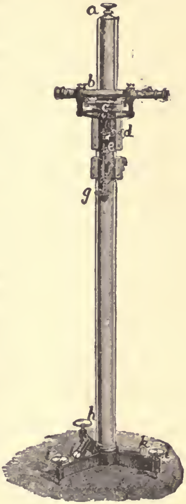

Fig. 286.

top of the shaft is to be inclined slightly in this direction. After a series of trials the axis may in this way be made perfectly vertical.

The object to be measured is to be set up with the aid of a plumb-line, beside a vertical scale, so as to be at the same distance from the cathetometer as the scale is, both at the top and at the bottom. The telescope of the cathetometer, accurately levelled, is to be focussed by means of the cross-hairs upon one end of the object (¶ 116, 3), then rotated so as to bear upon the scale, and the reading of the scale noted. If the spirit-level is disturbed, the cathetometer must be readjusted and the reading redetermined. The reading of the lower end of the object is to be found in the same way. By putting a graduated scale in place of the cross-hairs, the divisions of a scale may be divided into very small parts. This method is not so precise as that depending upon the use of a vernier or micrometer attached to the cathetometer, but may, in unskilled hands, give fully as accurate results.

¶ 263. **Micrometer Eye-Pieces.** — Instead of moving a telescope or a reading microscope bodily, as in ¶¶ 261 and 262, it is sometimes convenient to mount the cross-hairs upon a small slide within the eye-piece of an instrument, and to give a slow motion to the slide by means of a micrometer screw. The value of the micrometer divisions must be found for each instrument. A micrometer eye-piece gives indications much more precise than a fixed scale; but care must be taken not to alter the setting of an instrument by pressure upon the eye-piece in adjusting the micrometer, and, as in the dividing-engine (¶ 261), to turn the instrument always in a given direction up to a setting. If the micrometer is turned too far, it must be turned backward a considerable way, then forward to the desired point.[1]

In the best optical circles two microscopes with micrometer eye-pieces are usually provided. These are placed on opposite sides of the circle, in order that errors due to excentricity may be avoided.

¶ 264. **Regulators.** — For experiments involving the accurate measurement of time, a clock with a compensating pendulum, or a chronometer with a compensating balance is indispensable. The clock or chronometer should be provided with an electric break-circuit, and must be rated by observations with either a sextant (¶ 243) or a transit (see Pickering's Physical Manipulation, § 178), or by comparison with time signals from some observatory.

In the Physical Laboratory of Harvard College, the regulator employed is a common seconds-clock with a wooden pendulum-rod controlled by an electrical time circuit. The control consists simply of a fine spiral spring connecting the pendulum with the armature of a telegraph instrument in the circuit. Electrical signals, sent from the Astronomical Observatory at intervals of two seconds, are thus made to act mechanically upon the pendulum. When the latter has been carefully rated without the control, very small impulses are sufficient to prevent it from gaining or losing.

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[1] The "backlash" should be taken up, in so far as possible, by the action of a spring. Errors due to "backlash" may be thus greatly diminished, but not completely eliminated.

¶ 265. **Kater's Pendulum** (Fig. 287). — In Kater's form of reversible pendulum (see 249) the rod (*de*) is usually made of brass, a little over a metre long, 2 or 3 *cm.* wide and about 5 *mm.* thick. Two steel knife-edges, *bc* and *fg*, are attached firmly to this rod with a distance of about 1 metre between them. They are supported when the pendulum is in use, by agate planes, *b* and *c*. The bob (*h*) is a brass cylinder, weighing 1 or 2 kilograms. Movable counterpoises, *d* and *e*, serve to adjust the centre of oscillation. Two light and firm metallic pointers (*a* and *i*) may be used to magnify the oscillations.

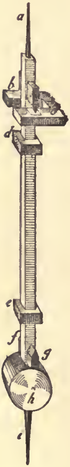

Fig. 287.

In addition to these adjustments, clamps with tangent-screws may be employed to obtain a slow motion of the counterpoises. The knife-edges *bc* and *fg* are sometimes made movable (one or both of them). In this case, verniers are usually attached, so that the distance between the knife-edges may be read by a scale on the shaft *de*. The zero-reading of the vernier is found by bringing the knife-edges together against a pressure equal to the whole weight of the pendulum. The accuracy of the main scale is tested by a comparator (¶ 260) at the ordinary temperature of the experiments, and under a strain equal to the average weight which the shaft sustains.

¶ 266. **Chronographs.** — A chronograph consists generally of a cylindrical drum (*A*, Fig. 288) rotated uniformly by clock-work. The surface of the drum is coated with lampblack, so that a style (*B*), attached to the armature (*c*) of a telegraph instrument may make a mark upon it. The line *AB* represents the trace caused by an ordinary seconds break-circuit. At the point *D* there is an extra break due to a signal given by hand. If the drum revolves uniformly, the exact time of such a break can evidently be determined by measuring the distance from it to the nearest second-mark, and comparing this with the distance between two second-marks.

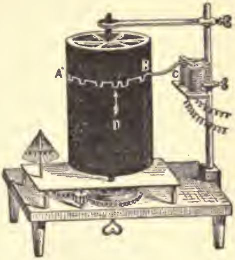

Fig. 288.

The pitch of a tuning-fork may be determined very exactly by the trace made on the surface of a chronograph (see ¶ 139).

It may be said in general that the chronograph is valuable as a means of determining precisely the interval of time between any two phenomena which, with or without the agency of electricity, are capable of affecting the motion of a style.

¶ 267. **The Siren.** — The siren (Fig. 289) is an instrument for producing a musical note of any pitch, and at the same time registering the number of vibrations constituting that note. It is operated by a constant air pressure from a bellows, specially constructed for this purpose. The air enters the wind-chest of the instrument at (F), issues obliquely from a series of holes (of which E is one) in the top of the wind-chest, and strikes obliquely against the sides of a series of holes (of which D is one) in a disc (C), which is thereby set in motion. When the two series of holes come opposite, the air escapes freely from the wind-chest; when they are not opposite, the current of air is nearly cut off. The irregular flow of the air sets the atmosphere in vibration. The number of vibrations in a given length of time is indicated by the dials A and B.

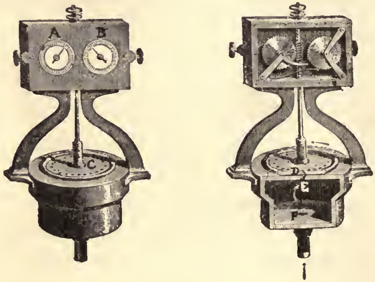

Fig. 289.

In practice the speed of the siren is regulated by pressure on the top of the bellows used to drive it. The note is slowly raised until it agrees with one whose pitch is to be determined. When the two notes are nearly in unison beats will be heard (¶ 140). By a slight change of air pressure, perfect unison may generally be obtained. This will be shown by a cessation of beats. The unison is maintained for a given length of time during which the number of vibrations made by the siren is registered. In some instruments the dials may be thrown in and out of gear at a given moment. This facilitates the observations of the dials, but care must be taken that the speed of the siren is not affected.

It must be remembered that beats occur not only when two notes are in unison, but also when they are nearly an octave apart, and to a somewhat less extent, when they are separated by any other musical interval (¶ 134). A musical ear is therefore almost a necessity in the adjustment of a siren. The chief advantage of the siren is that it enables us to find the pitch of notes not easily determined (as is Exps. 52, 54, and 55), by either optical or graphical methods.

¶ 268. **Mirror Galvanometers.** — A very sensitive galvanometer is made by suspending a small mirror (*F*, Fig. 290) in the middle of a coil *E* of insulated wire, by means of a single fibre of cocoon silk (*DE*). Small bits of "hair-spring" (used in watches) highly magnetized, all in the same manner, are fastened with the smallest possible quantity of wax to the back of the mirror. A large curved magnet (*BC*) capable of sliding up and down the tube (*A*) or turning round it, is adjusted so as to nearly neutralize the effect of the earth's magnetism on the magnets attached to the

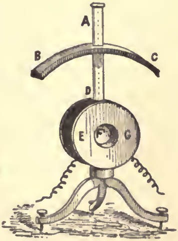

Fig. 290.

mirror. The sensitiveness of this instrument when accurately adjusted, though less permanent than that of an astatic combination, is for the time being fully as great.

In some galvanometers a converging mirror is used, so that a spot of light may be projected on a transparent screen. The existence of a current is indicated by the motion of the spot of light with respect to a scale graduated on the screen.

In other instruments a plane mirror is employed, with a long-focus lens mounted permanently in front of it. The deflection of the mirror is frequently observed by means of the reflection (*E*, Fig. 291) of a scale (*B*) in the mirror (*C*), seen from a point (*A*), where either the eye or a telescope may be placed.[1]

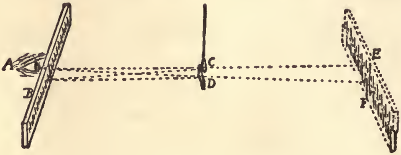

FIG. 291.

¶ 269. **Electrical Standards.** — Copies of "standard ohms" may be obtained from most dealers in electrical apparatus. The terminals should be thick copper rods, capable of being amalgamated with mercury and connected by mercury cups with a Wheatstone's Bridge Apparatus. Unless special care be taken in making these connections, the most accurate standards of resistance may lead to very erroneous results.

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[1] Prof. B. O. Peirce has shown that excellent results may be obtained without any telescope (*A*), by placing beneath the mirror *C* a fixed mirror *D*, so that the two reflections (*E* and *F*) of the scale (*B*) very nearly coincide. When the two mirrors are parallel, the zeros of the two scales are opposite, *no matter where the eye may be placed.* The slightest deflection of the mirror causes an apparent motion of the scale reflected in it.

Standard cells of Latimer Clark's pattern may easily be obtained. Their electromotive force is about 1·435 volts at 15°. The decrease is about ·00077 volts for a rise of temperature of 1° Centigrade. The uses of a constant cell have been alluded to in ¶¶ 228, 230.

"Standard ampères" are now being made by some dealers. When the attraction of a coil of wire for a piece of soft iron is balanced by gravity (Fig. 292), an allowance must be made for variations in gravity when the instrument is transported from one latitude to another. A standard ampère depending upon the action of a spring, though subject to many theoretical objections, would be practically useful as a check upon results obtained by other methods. Let us suppose that such an instrument is connected in series with a rheostat and a tangent galvanometer, that a current, sent through both, is increased until the instrument indicates 1 ampère, and that the galvanometer is then read. The reciprocal of the tangent

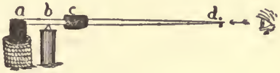

Fig. 292.

of the angle of deflection should agree closely with the reduction factor already found (Exp. 83).

¶ 270. **Electrometers.** — Various forms of quadrant electrometer may now be obtained from manufacturers. The theory of these instruments is exceedingly complicated, and the results are more or less uncertain. The principal use of the instrument is in the case of inconstant cells, to confirm results obtained by the use of a condenser. Such instruments in general have to be calibrated by means of cells of known electromotive force.

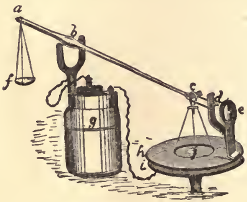

FIG 293.

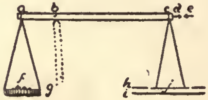

FIG. 294.

Thomson's absolute electrometer (Figs. 293 and 294) depends upon the attraction between two plates *j* and *i*, when charged oppositely with electricity. The plate *j* is suspended from one end (*c*) of a balance-beam (*ac*). The force exerted upon it is counterpoised by weights in a pan (*f*) suspended from the other end of the beam (*a*.) The deflection of the beam is observed by means of a sight (*d*) and a lens (*e*). The plate *i* is very much larger than *j*, which is surrounded by a ring (*h*) charged to the same potential as the movable disk (*j*), to equalize the distribution of electricity upon the latter.

If *w* is the weight required to balance the attraction of the two plates, *d* the distance between them, and *a* the area of the suspended plate (*j*), then the difference of potential (*e*) between the plates is given in electrostatic measure by the formula —

$$e = d\sqrt{\frac{8\pi g w}{a}}.$$

It is said that an absolute electrometer may be made sensitive to the difference in potential between the two poles of a Daniell cell. It is especially valuable for the calibration of other forms of electrometer better suited for actual use, and for determinations of the fundamental relations between the electrostatic and electro-magnetic systems.

END OF PART II.

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